ECEN 667
Power System Stability

Lecture 22: Small Signal Stability

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

Announcements

- Read Chapter 8
- Homework 5 is due on Thursday Nov 14

Small Signal Stability Analysis

- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
  - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) obviously must be stable
- Small system stability analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
  - There must be positive damping

Model Based SSA

- Assume the power system is modeled in our standard form as

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{y}\right)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

- The system can be linearized about an equilibrium point

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{y}$$

$$\mathbf{0} = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta\mathbf{y}$$

If there are just classical generator models then **D** is the power flow Jacobian; otherwise it also includes the stator algebraic equations

- Eliminating $\Delta\mathbf{y}$ gives

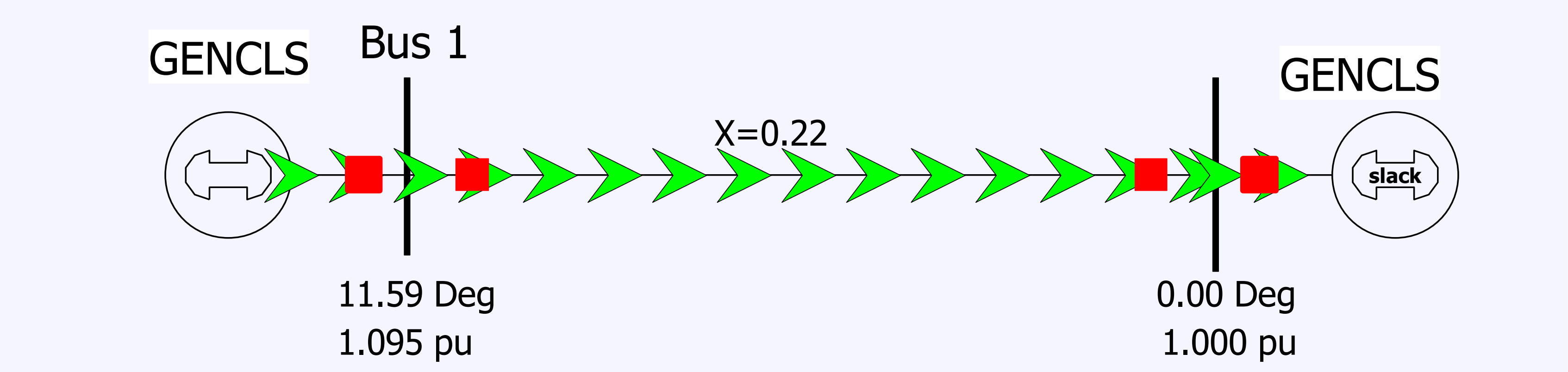
$$\Delta\dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)\Delta\mathbf{x} = \mathbf{A}_{\mathbf{sys}}\Delta\mathbf{x}$$

Model Based SSA

- The matrix $\mathbf{A}_{sys}$ can be calculated doing a partial factorization, just like what was done with Kron reduction (Ward equivalents)
- SSA is done by looking at the eigenvalues (and other properties) of $\mathbf{A}_{sys}$

SSA Two Generator Example

- Consider the two bus, two classical generator system from lectures 18 and 20 with $X_{d1}'=0.3$, $H_1=3.0$, $X_{d2}'=0.2$, $H_2=6.0$



- Essentially everything needed to calculate the **A**, **B**, **C** and **D** matrices was covered in lecture 19

SSA Two Generator Example

- The **A** matrix is calculated differentiating **f**(**x**,**y**) with respect to **x** (where **x** is δ_1, $\Delta\omega_1$, δ_2, $\Delta\omega_2$)

$$\frac{d\delta_1}{dt} = \Delta\omega_{1.pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1}\left(P_{M1} - P_{E1} - D_1\Delta\omega_{1.pu}\right)$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2.pu}\omega_s$$

$$\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2}\left(P_{M2} - P_{E2} - D_2\Delta\omega_{1.pu}\right)$$

$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

$$E_{Di} + jE_{Qi} = E_i'\left(\cos\delta_i + j\sin\delta_i\right)$$

SSA Two Generator Example

- Giving

$$\mathbf{A} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.761 & 0 & 0 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0 & 0 & -0.389 & 0 \end{bmatrix}$$

- **B**, **C** and **D** are as calculated previously for the implicit integration, except the elements in B are not multiplied by Δt/2

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.2889 & 0.6505 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0.3893 \end{bmatrix}$$

SSA Two Generator Example

- The **C** and **D** matrices are

$$\mathbf{C} = \begin{bmatrix} -3.903 & 0 & 0 & 0 \\ -1.733 & 0 & 0 & 0 \\ 0 & 0 & -4.671 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 7.88 & 0 & -4.54 \\ -7.88 & 0 & 4.54 & 0 \\ 0 & -4.54 & 0 & 9.54 \\ 4.54 & 0 & -9.54 & 0 \end{bmatrix}$$

- Giving

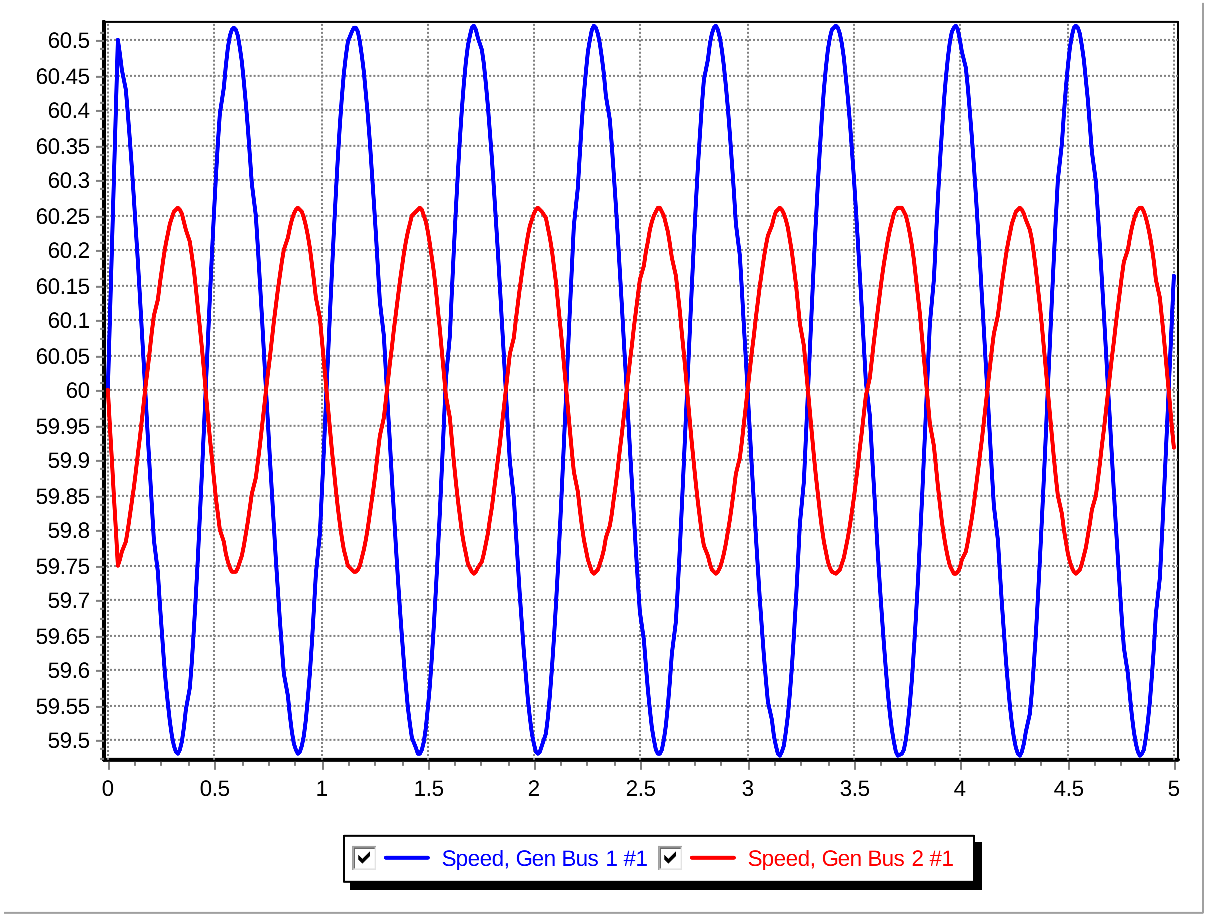
$$\mathbf{A}_{sys} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.229 & 0 & 0.229 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0.114 & 0 & -0.114 & 0 \end{bmatrix}$$

SSA Two Generator

- Calculating the eigenvalues gives a complex pair and two zero eigenvalues
- The complex pair, with values of +/- j11.39 corresponds to the generators oscillating against each other at 1.81 Hz
- One of the zero eigenvalues corresponds to the lack of an angle reference
  - Could be rectified by redefining angles to be with respect to a reference angle (see book 226) or we just live with the zero
- Other zero is associated with lack of speed dependence in the generator torques

SSA Two Generator Speeds

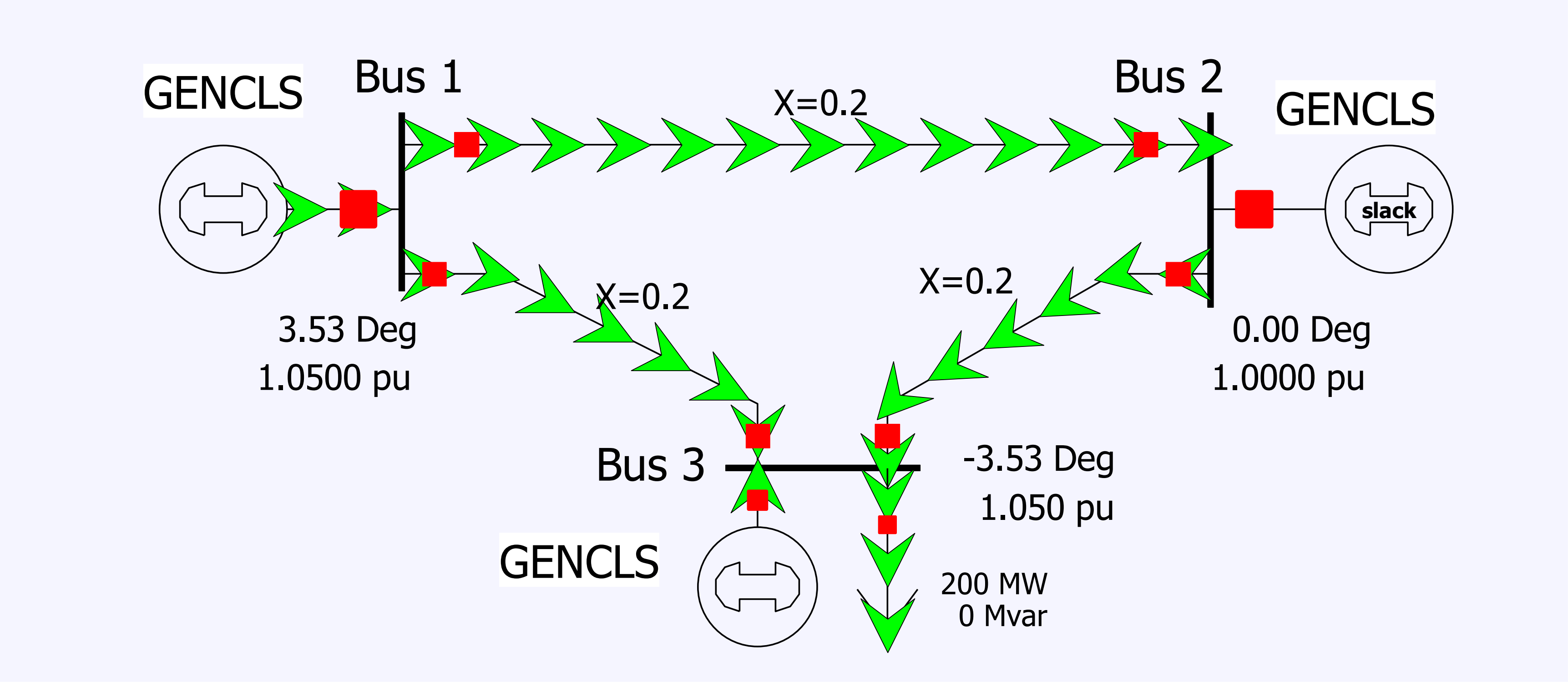
- The two generator system response is shown below for a small disturbance



Notice the actual response closely matches the calculated frequency

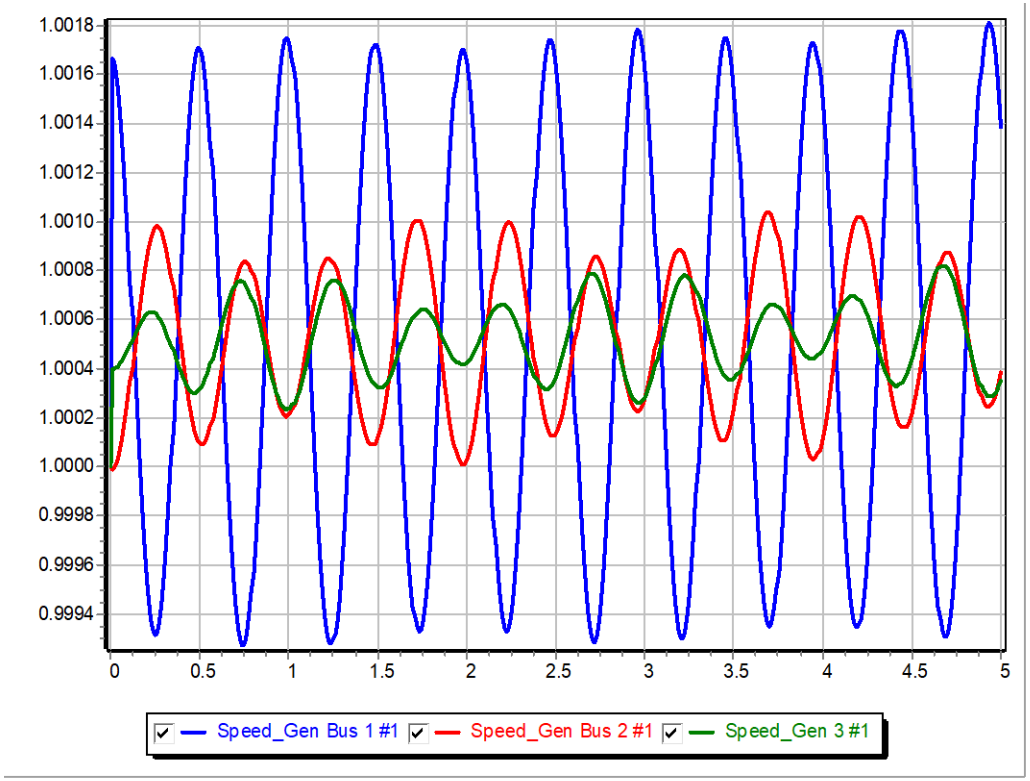
SSA Three Generator Example

- The two generator system is extended to three generators with the third generator having H_3 of 8 and $X_{d3}'=0.3$



SSA Three Generator Example

- Using SSA, two frequencies are identified: one at 2.02 Hz and one at 1.51 Hz



The oscillation is started with a short, self-clearing fault

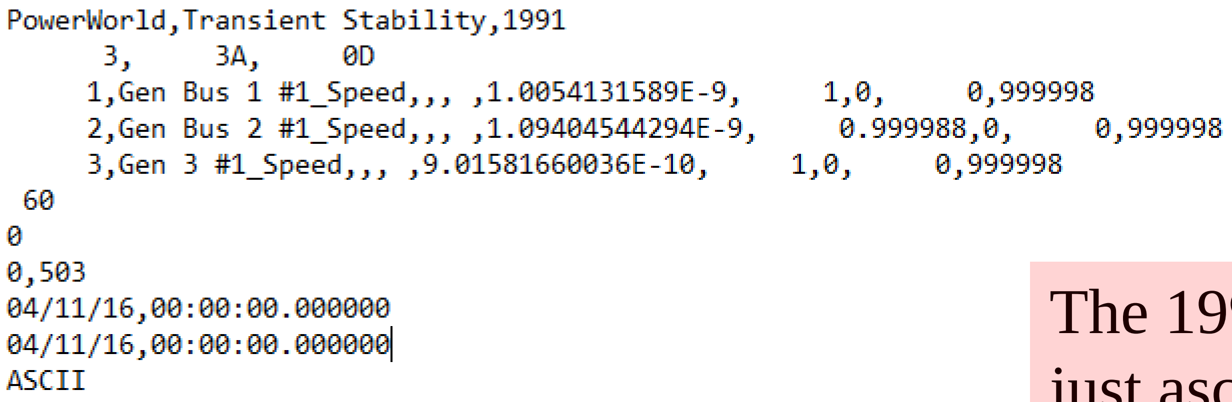
Shortly we'll discuss modal analysis to determine the contribution of each mode to each signal

PowerWorld case **B2_CLS_3Gen_SSA**

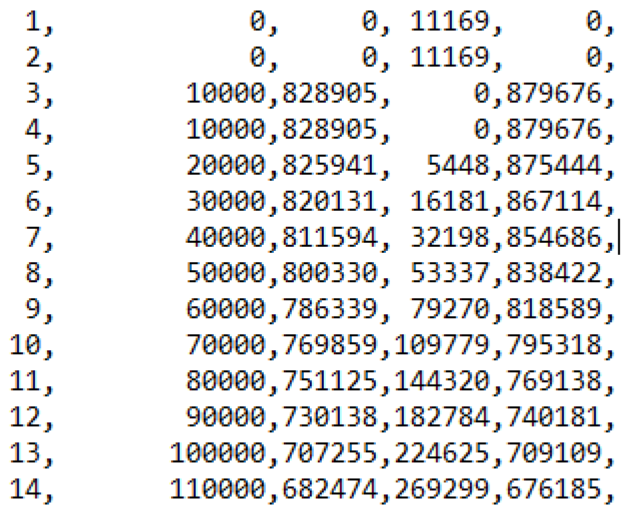
Comtrade Format (IEEE Std. C37.111)

- Comtrade is a standard for exchanging power system time-varying data
  - Originally developed for power system transient results such as from digital fault recorders (DFRs), but it can be used for any data
- Comtrade is now being used for the exchange of PMU data and transient stability results
- Three variations on the standard (1991, 1999 and 2013 format)
- PowerWorld now allows transient stability results to be quickly saved in all three Comtrade Formats

Three Bus Example Comtrade Results



```
PowerWorld,Transient Stability,1991
      3,     3A,     0D
     1,Gen Bus 1 #1_Speed,,, ,1.0054131589E-9,     1,0,     0,999998
     2,Gen Bus 2 #1_Speed,,, ,1.09404544294E-9,     0.999988,0,     0,999998
     3,Gen 3 #1_Speed,,, ,9.01581660036E-10,     1,0,     0,999998
 60
0
0,503
04/11/16,00:00:00.000000
04/11/16,00:00:00.000000
ASCII
```



```
         1,           0,     0, 11169,     0,
         2,           0,     0, 11169,     0,
         3,       10000,828905,     0,879676,
         4,       10000,828905,     0,879676,
         5,       20000,825941,  5448,875444,
         6,       30000,820131, 16181,867114,
         7,       40000,811594, 32198,854686,
         8,       50000,800330, 53337,838422,
         9,       60000,786339, 79270,818589,
        10,       70000,769859,109779,795318,
        11,       80000,751125,144320,769138,
        12,       90000,730138,182784,740181,
        13,      100000,707255,224625,709109,
        14,      110000,682474,269299,676185,
```

The 1991 format is just ascii using four files; the 1999 format extends to allow data to be stored in binary format; the 2013 format extends to allow a single file format

Large System Studies

- The challenge with large systems, which could have more than 100,000 states, is the shear size
  - Most eigenvalues are associated with the local plants
  - Computing all the eigenvalues is computationally challenging, order n^3
- Specialized approaches can be used to calculate particular eigenvalues of large matrices
  - See Kundur, Section 12.8 and associated references

Relationship to Signal-Based Modal Analysis

- Both model-based and signal-based modal analysis are trying to get similar information
- The advantage of the signal-based approach is it does not need a model and does not require calculating the eigenvalues of potentially quite large matrices
- Disadvantages are lack of sensitivity information and the inability to see many modes
- The model-based approach can potentially provide more information, but the quality of the information depends on the model
- Disadvantages are need to deal with potentially quite large matrices and the need for a model.

Single Machine Infinite Bus

- A quite useful analysis technique is to consider the small signal stability associated with a single generator connected to the rest of the system through an equivalent transmission line
- Driving point impedance looking into the system is used to calculate the equivalent line's impedance
  - The Z_{ii} value can be calculated quite quickly using sparse vector methods (a fast forward and fast backward)
- Rest of the system is assumed to be an infinite bus with its voltage set to match the generator's real and reactive power injection and voltage

Small SMIB Example

- The infinite bus voltage is then calculated so as to match the bus i terminal voltage and current

$$\bar{V}_{\text{inf}} = \bar{V}_i - Z_i \bar{I}_i$$

$$\text{where } \left(\frac{P_i + jQ_i}{\bar{V}_i} \right)^* = \bar{I}_i$$

While this was demonstrated on an extremely small system for clarity, the approach works the same for any sized system

- In the example we have

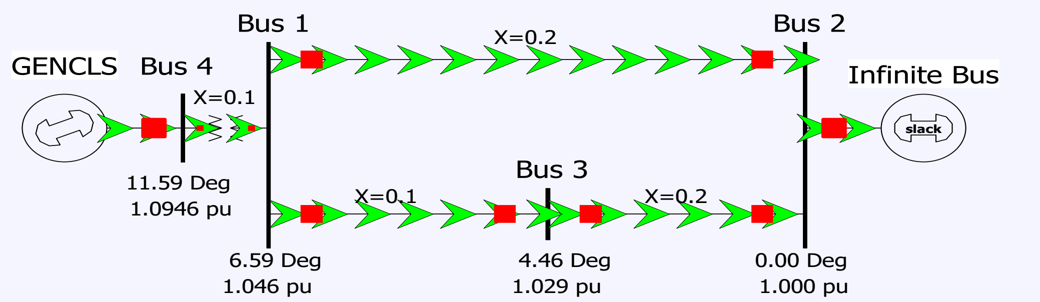
$$\left(\frac{P_4 + jQ_4}{\bar{V}_4} \right)^* = \left(\frac{1 + j0.572}{1.072 + j0.220} \right)^* = 1 - j0.328$$

$$\bar{V}_{\text{inf}} = (1.072 + j0.220) - (j0.22)(1 - j0.328)$$

$$\bar{V}_{\text{inf}} = 1.0$$

Small SMIB Example

- As a small example, consider the 4 bus system shown below, in which bus 2 really is an infinite bus



- To get the SMIB for bus 4, first calculate Z_{44}

$$\mathbf{Y}_{\mathbf{bus}} = j\begin{bmatrix} -25 & 0 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 10 & 0 & -15 & 0 \\ 10 & 0 & 0 & -13.33 \end{bmatrix} \rightarrow Z_{44} = j0.1269$$

Z_{44} is Z_{th} in parallel with $jX'_{d,4}$ (which is j0.3) so Z_{th} is j0.22

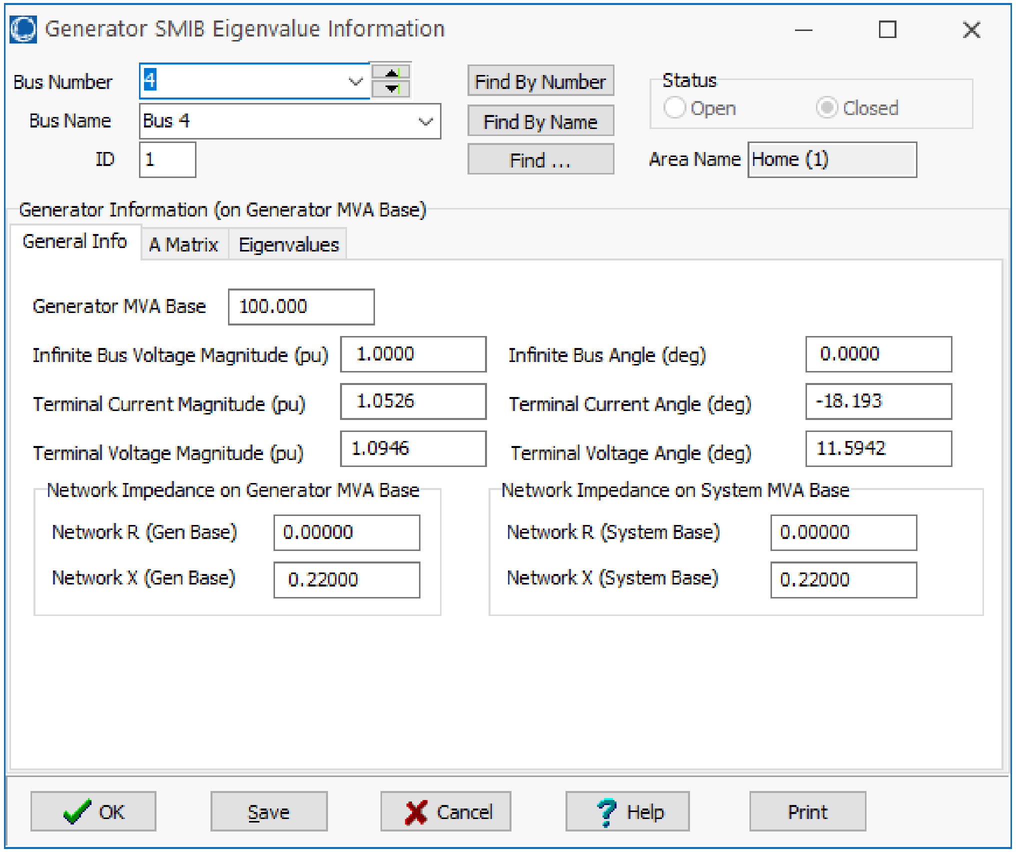
Here I retained the infinite bus, but replaced its column and row by zeros except 1 on the diagonal

Calculating the A Matrix

- The SMIB model **A** matrix can then be calculated either analytically or numerically
  - The equivalent line's impedance can be embedded in the generator model so the infinite bus looks like the "terminal"
- This matrix is calculated in PowerWorld by selecting **Transient Stability, SMIB Eigenvalues**
  - Select **Run SMIB Eigen Analysis** to perform an SMIB analysis for all the generators in a case
  - Right click on a generator on the SMIB form and select **Show SMIB Dialog** to see the Generator SMIB Eigenvalue Dialog
  - These two bus equivalent networks can also be saved, which is useful for understanding the behavior of individual generators

Example: Bus 4 SMIB Dialog

- On the SMIB dialog, the General Information tab shows information about the two bus equivalent



Generator SMIB Eigenvalue Information

Bus Number	4
Bus Name	Bus 4
ID	1
Status	○ Open ● Closed
Area Name	Home (1)

Generator Information (on Generator MVA Base)

Generator MVA Base	100.000		
Infinite Bus Voltage Magnitude (pu)	1.0000	Infinite Bus Angle (deg)	0.0000
Terminal Current Magnitude (pu)	1.0526	Terminal Current Angle (deg)	-18.193
Terminal Voltage Magnitude (pu)	1.0946	Terminal Voltage Angle (deg)	11.5942

Network Impedance on Generator MVA Base

Network R (Gen Base)	0.00000
Network X (Gen Base)	0.22000

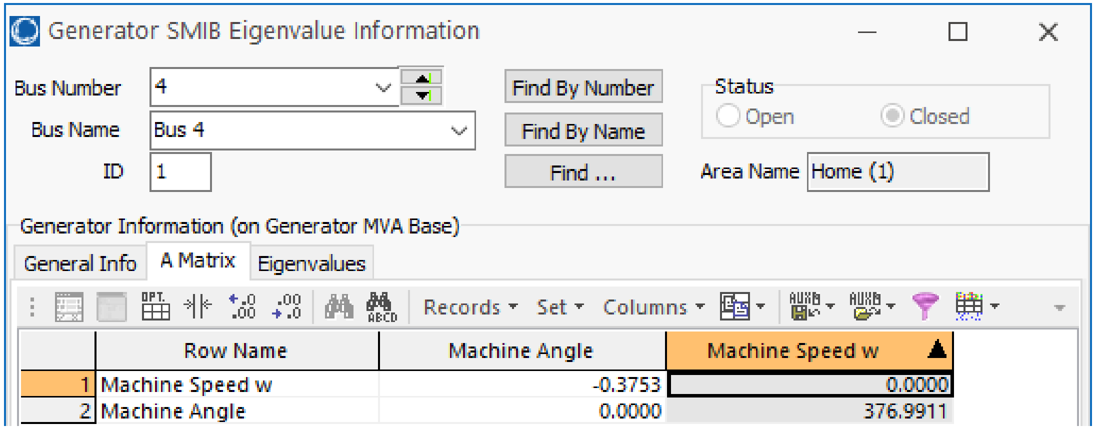
Network Impedance on System MVA Base

Network R (System Base)	0.00000
Network X (System Base)	0.22000

PowerWorld case **B4_SMIB**

Example: Bus 4 SMIB Dialog

- On the SMIB dialog, the **A** Matrix tab shows the $\mathbf{A}_{sys}$ matrix for the SMIB generator



Generator SMIB Eigenvalue Information

Bus Number: 4

Bus Name: Bus 4

ID: 1

Status: ○ Open ● Closed

Area Name: Home (1)

Generator Information (on Generator MVA Base)

General Info | A Matrix | Eigenvalues

	Row Name	Machine Angle	Machine Speed w
1	Machine Speed w	-0.3753	0.0000
2	Machine Angle	0.0000	376.9911

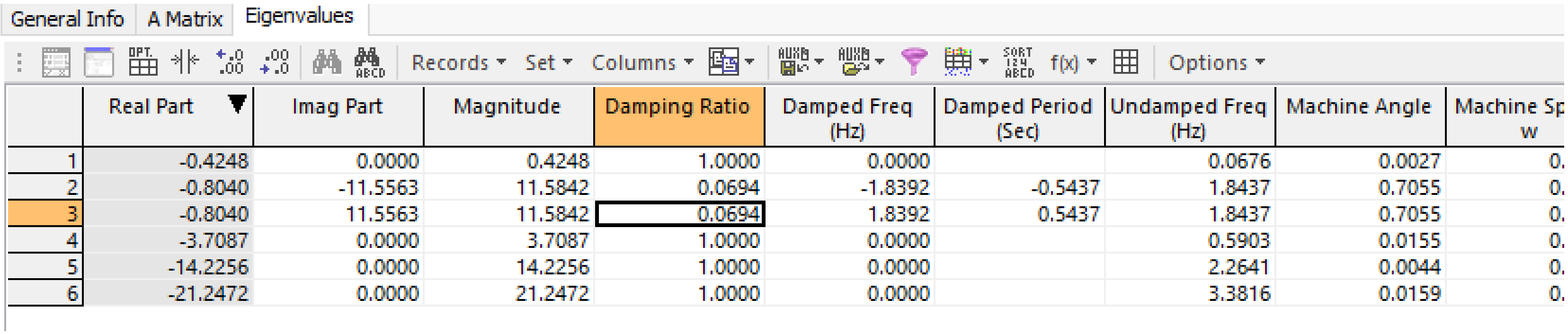
- In this example A_{21} is showing

$$\frac{\partial \Delta\omega_{4,pu}}{\partial\delta_4} = \frac{1}{2H_4}\left(\frac{-\partial P_{E,4}}{\partial\delta_4}\right) = -\left(\frac{1}{6}\right)\left(\left(\frac{-1}{0.3+0.22}\right)\left(-1.2812\cos\left(23.94^\circ\right)\right)\right)$$

$$= -0.3753$$

Example: Bus 4 with GENROU Model

- The eigenvalues can be calculated for any set of generator models
- The example can be extended by replacing the bus 4 generator classical machine with a GENROU model
  - There are now six eigenvalues, with the dominate response coming from the electro-mechanical mode with a frequency of 1.84 Hz, and damping of 6.9%



	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Sp w
1	-0.4248	0.0000	0.4248	1.0000	0.0000		0.0676	0.0027	0.
2	-0.8040	-11.5563	11.5842	0.0694	-1.8392	-0.5437	1.8437	0.7055	0.
3	-0.8040	11.5563	11.5842	0.0694	1.8392	0.5437	1.8437	0.7055	0.
4	-3.7087	0.0000	3.7087	1.0000	0.0000		0.5903	0.0155	0.
5	-14.2256	0.0000	14.2256	1.0000	0.0000		2.2641	0.0044	0.
6	-21.2472	0.0000	21.2472	1.0000	0.0000		3.3816	0.0159	0.

PowerWorld case **B4_SMIB_GENROU**