Announcements

• Read Chapter 7
• Exam average: 86; high of 100
• Homework 4 is assigned today. It is due on Tuesday Oct 29
Adding Network Equations

• Previous slides with the network equations embedded in the differential equations were a special case.

• In general with the explicit approach we'll be alternating between solving the differential equations and solving the algebraic equations.

• Voltages and currents in the network reference frame can be expressed using either polar or rectangular coordinates.

• In rectangular with the book's notation we have

\[
\vec{V}_i = V_{Di} + jV_{Qi}, \quad \vec{I}_i = I_{Di} + jI_{Qi}
\]
Adding Network Equations

- Network equations will be written as $Y \cdot V - I(x,V) = 0$
  - Here $Y$ is as from the power flow, except augmented to include the impact of the generator's internal impedance.
  - Constant impedance loads are also embedded in $Y$; non-constant impedance loads are included in $I(x,V)$.

- If $I$ is independent of $V$ then this can be solved directly: $V = Y^{-1}I(x)$

- In general an iterative solution is required, which we'll cover shortly, but initially we'll go with just the direct solution.
Two Bus Example, Except with No Infinite Bus

- To introduce the inclusion of the network equations, the previous example is extended by replacing the infinite bus at bus 2 with a classical model with $X_{d2}'=0.2$, $H_2=6.0$

PowerWorld Case is **B2_CLS_2Gen**
Bus Admittance Matrix

- The network admittance matrix is

\[ Y_N = \begin{bmatrix} -j4.545 & j4.545 \\ j4.545 & -j4.545 \end{bmatrix} \]

- This is augmented to represent the Norton admittances associated with the generator models (\(X_{d1}'=0.3\), \(X_{d2}'=0.2\))

\[ Y = Y_N + \begin{bmatrix} \frac{1}{j0.3} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix} \]

In PowerWorld you can see this matrix by selecting **Transient Stability, States/Manual Control, Transient Stability Ybus**
Current Vector

- For the classical model the Norton currents are given by
  \[ \bar{I}_{Ni} = \frac{E'_i \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}} \]

- The initial values of the currents come from the power flow solution

- As the states change (\(\delta_i\) for the classical model), the Norton current injections also change
### B2_CLS_Gen Initial Values

- The internal voltage for generator 1 is as before
  \[ \bar{I} = 1 - j0.3286 \]
  \[ \bar{E}_1 = 1.0 + (j0.22 + j0.3)\bar{I} = 1.1709 + j0.52 = 1.281 \angle 23.95^\circ \]

- We likewise solve for the generator 2 internal voltage
  \[ \bar{E}_2 = 1.0 - (j0.2)\bar{I} = 0.9343 - j0.2 = 0.9554 \angle -12.08^\circ \]

- The Norton current injections are then
  \[ \bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903 \]
  \[ \bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714 \]

Keep in mind the Norton current injections are not the current out of the generator.
To check the values, solve for the voltages, with the values matching the power flow values

\[
V = \begin{bmatrix}
-j7.879 & j4.545 \\
 j4.545 & -j9.545
\end{bmatrix}^{-1} \begin{bmatrix}
1.733 - j3.903 \\
-1 - j4.671
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1.072 + j0.22 \\
1.0
\end{bmatrix}
\]
Swing Equations

- With the network constraints modeled, the swing equations are modified to represent the electrical power in terms of the generator's state and current values

\[ P_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi} \]

- Then swing equation is then

\[
\frac{d\delta_i}{dt} = \Delta \omega_{i,pu} \omega_s \\
\frac{d\Delta \omega_{i,pu}}{dt} = \frac{1}{2H_i} \left( P_{Mi} - \left( E_{Di} I_{Di} + E_{Qi} I_{Qi} \right) - D_i \left( \Delta \omega_{i,pu} \right) \right)
\]

\[ I_{Di} + jI_{Qi} \] is the current being injected into the network by the generator.
Two Bus, Two Generator Differential Equations

- The differential equations for the two generators are

\[
\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s
\]

\[
\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1}\left(P_{M1} - \left(E_{D1}I_{D1} + E_{Q1}I_{Q1}\right)\right)
\]

\[
\frac{d\delta_2}{dt} = \Delta\omega_{2,pu}\omega_s
\]

\[
\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2}\left(P_{M2} - \left(E_{D2}I_{D2} + E_{Q2}I_{Q2}\right)\right)
\]

In this example
\[P_{M1} = 1\] and \[P_{M2} = -1\]
PowerWorld GENCLS Initial States
Solution at t=0.02

• Usually a time step begins by solving the differential equations. However, in the case of an event, such as the solid fault at the terminal of bus 1, the network equations need to be first solved.

• Solid faults can be simulated by adding a large shunt at the fault location.
  – Amount is somewhat arbitrary, it just needs to be large enough to drive the faulted bus voltage to zero.

• With Euler's the solution after the first time step is found by first solving the differential equations, then resolving the network equations.
Solution at t=0.02

- Using $Y_{\text{fault}} = -j1000$, the fault-on conditions become

$$V = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.733 - j3.903 \\ -1 - j4.671 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.1053 \end{bmatrix}$$

Solving for the currents into the network

$$I_1 = \frac{(1.1702 + j0.52) - V_1}{j0.3} = 1.733 - j3.900$$

$$I_2 = \frac{(0.9343 - j0.2) - (0.486 - j0.1053)}{j0.2} = -0.473 - j2.240$$
Solution at t=0.02

• Then the differential equations are evaluated, using the new voltages and currents
  – These impact the calculation of $P_{Ei}$ with $P_{E1}=0$, $P_{E2}=0$

\[
\begin{bmatrix}
\delta_1(0.02) \\
\Delta \omega_1(0.02) \\
\delta_2(0.02) \\
\Delta \omega_1(0.02)
\end{bmatrix}
= \begin{bmatrix}
0.418 \\
0.0 \\
-0.211 \\
0
\end{bmatrix} + 0.02 \begin{bmatrix}
0 \\
\frac{1}{6}(1-0) \\
0 \\
\frac{1}{12}(-1-0)
\end{bmatrix}
= \begin{bmatrix}
0.418 \\
0.00333 \\
-0.211 \\
-0.00167
\end{bmatrix}
\]

• If solving with Euler's this is the final state value; using these state values the network equations are resolved, with the solution the same here since the $\delta$'s didn't vary
PowerWorld GENCLS at t=0.02
Solution Values Using Euler's

• The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

<table>
<thead>
<tr>
<th>Time (Sec)</th>
<th>Gen 1 Rotor Angle</th>
<th>Gen 1 Speed (Hz)</th>
<th>Gen 2 Rotor Angle</th>
<th>Gen2 Speed (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23.9462</td>
<td>60</td>
<td>-12.0829</td>
<td>60</td>
</tr>
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</table>
Solution at $t=0.02$ with RK2

- With RK2 the first part of the time step is the same as Euler's, that is solving the network equations with

$$x(t + \Delta t)^{(1)} = x(t) + k_1 = x(t) + \Delta T f(x(t))$$

- Then calculate $k_2$ and get a final value for $x(t+\Delta t)$

$$k_2 = \Delta t \ f \left( x(t) + k_1 \right)$$

$$x(t + \Delta t) = x(t) + \frac{1}{2} \left( k_1 + k_2 \right)$$

- Finally solve the network equations using the final value for $x(t+\Delta t)$
Solution at $t=0.02$ with RK2

- From the first half of the time step

$$x(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- Then

$$k_2 = \Delta t \ f \left( x(t) + k_1 \right) = 0.02 \begin{bmatrix} 1.256 \\ \frac{1}{6}(1-0) \\ -0.628 \\ \frac{1}{12}(-1-0) \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \\ -0.0126 \\ -0.00167 \end{bmatrix}$$
Solution at $t=0.02$ with RK2

- The new values for the Norton currents are

$$\bar{I}_{N1} = \frac{1.281 \angle 24.69^\circ}{j0.3} = 1.851 - j3.880$$

$$\bar{I}_{N2} = \frac{0.9554 \angle -12.43^\circ}{j0.2} = -1.028 - j4.665$$

$$\mathbf{V}(0.02) = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.851 - j3.880 \\ -1.028 - j4.665 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.108 \end{bmatrix}$$
Solution Values Using RK2

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

<table>
<thead>
<tr>
<th>Time (Sec)</th>
<th>Gen 1 Rotor Angle</th>
<th>Gen 1 Speed (Hz)</th>
<th>Gen 2 Rotor Angle</th>
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<tbody>
<tr>
<td>0</td>
<td>23.9462</td>
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</tr>
</tbody>
</table>
Angle Reference

- The initial angles are given by the angles from the power flow, which are based on the slack bus's angle.
- As presented the transient stability angles are with respect to a synchronous reference frame.
  - Sometimes this is fine, such as for either shorter studies, or ones in which there is little speed variation.
  - Oftentimes this is not best since the when the frequencies are not nominal, the angles shift from the reference frame.
- Other reference frames can be used, such as with respect to a particular generator's value, which mimics the power flow approach; the selected reference has no impact on the solution.
Subtransient Models

- The Norton current injection approach is what is commonly used with subtransient models in industry.
- If subtransient saliency is neglected (as is the case with GENROU and GENSAL in which $X''_d=X''_q$) then the current injection is

$$I_{Nd} + jI_{Nq} = \frac{E'_d + jE'_q}{R_s + jX''} = \frac{(-\psi'_q + j\psi'_d) \omega}{R_s + jX''}$$

- Subtransient saliency can be handled with this approach, but it is more involved (see Arrillaga, *Computer Analysis of Power Systems*, section 6.6.3)
Subtransient Models

- Note, the values here are on the dq reference frame
- We can now extend the approach introduced for the classical machine model to subtransient models
- Initialization is as before, which gives the δ's and other state values
- Each time step is as before, except we use the δ's for each generator to transfer values between the network reference frame and each machine's dq reference frame
  - The currents provide the coupling
Two Bus Example with Two GENROU Machine Models

- Use the same system as before, except with we'll model both generators using GENROUs
  - For simplicity we'll make both generators identical except set $H_1=3$, $H_2=6$; other values are $X_d=2.1$, $X_q=0.5$, $X'_d=0.2$, $X'_q=0.5$, $X''_q=X''_d=0.18$, $X_l=0.15$, $T'_d=7.0$, $T'_q=0.75$, $T''_d=0.035$, $T''_q=0.05$; no saturation
  - With no saturation the value of the $\delta$'s are determined (as per the earlier lectures) by solving
    \[
    |E| \angle \delta = V + (R_s + jX_q)I
    \]
  - Hence for generator 1
    \[
    |E_1| \angle \delta_1 = 1.0946 \angle 11.59^\circ + (j0.5)(1.052 \angle -18.2^\circ) = 1.431 \angle 30.2^\circ
    \]
GENROU Block Diagram
Two Bus Example with Two GENROU Machine Models

- Using the early approach the initial state vector is

\[
\begin{bmatrix}
\delta_1 \\
\Delta \omega_1 \\
E'_{q1} \\
\psi_{1d1} \\
\psi_{2q1} \\
E'_{d1} \\
\delta_2 \\
\Delta \omega_2 \\
E'_{q2} \\
\psi_{1d2} \\
\psi_{2q2} \\
E'_{d2}
\end{bmatrix} =
\begin{bmatrix}
0.5273 \\
0.0 \\
1.1948 \\
1.1554 \\
0.2446 \\
0 \\
-0.5392 \\
0 \\
0.9044 \\
0.8928 \\
-0.3594 \\
0
\end{bmatrix}
\]

Note that this is a salient pole machine with \( X'_q = X_q \); hence \( E'_d \) will always be zero.

The initial currents in the dq reference frame are

\( I_{d1} = 0.7872, I_{q1} = 0.6988, I_{d2} = 0.2314, I_{q2} = -1.0269 \)

Initial values of \( \psi''_{q1} = -0.2236, \) and \( \psi''_{d1} = 1.179 \)
Solving with Euler's

- We'll again solve with Euler's, except with $\Delta t$ set now to 0.01 seconds (because now we have a subtransient model with faster dynamics)
  - We'll also clear the fault at $t=0.05$ seconds
- For the more accurate subtransient models the swing equation is written in terms of the torques

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta \omega_i$$

$$\frac{2H_i}{\omega_s} \frac{d\omega_i}{dt} = \frac{2H_i}{\omega_s} \frac{d\Delta \omega_i}{dt} = T_{Mi} - T_{Ei} - D_i \left( \Delta \omega_i \right)$$

with $T_{Ei} = \psi''_{d,i} q_i - \psi''_{q,i} i_d$
Norton Equivalent Current Injections

- The initial Norton equivalent current injections on the dq base for each machine are

\[ I_{Nd1} + jI_{Nq1} = \frac{\left(-\psi''_q + j\psi''_d\right)\omega_1}{jX''_1} = \frac{(-0.2236 + j1.179)(1.0)}{j0.18} \]

\[ = 6.55 + j1.242 \]

\[ I_{ND1} + jI_{NQ1} = 2.222 - j6.286 \]

\[ I_{Nd2} + jI_{Nq2} = 4.999 + j1.826 \]

\[ I_{ND2} + jI_{NQ2} = -1 - j5.227 \]

Recall the dq values are on the machine's reference frame and the DQ values are on the system reference frame.
Moving between DQ and dq

- Recall

\[
\begin{bmatrix}
I_{di} \\
I_{qi}
\end{bmatrix} = \begin{bmatrix}
\sin \delta & -\cos \delta \\
\cos \delta & \sin \delta
\end{bmatrix} \begin{bmatrix}
I_{Di} \\
I_{Qi}
\end{bmatrix}
\]

- And

\[
\begin{bmatrix}
I_{Di} \\
I_{Qi}
\end{bmatrix} = \begin{bmatrix}
\sin \delta & \cos \delta \\
-\cos \delta & \sin \delta
\end{bmatrix} \begin{bmatrix}
I_{di} \\
I_{qi}
\end{bmatrix}
\]

The currents provide the key coupling between the two reference frames.
Bus Admittance Matrix

- The bus admittance matrix is as from before for the classical models, except the diagonal elements are augmented using

\[ Y_i = \frac{1}{R_{s,i} + jX''_{d,i}} \]

\[
Y = Y_N + \begin{bmatrix} \frac{1}{j0.18} & 0 \\ 0 & \frac{1}{j0.18} \end{bmatrix} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}
\]
Algebraic Solution Verification

- To check the values solve (in the network reference frame)

\[
V = \begin{bmatrix}
-j10.101 & j4.545 \\
-j4.545 & -j10.101 \\
\end{bmatrix}^{-1} \begin{bmatrix}
2.222 - j6.286 \\
-1 - j5.227 \\
\end{bmatrix} \\
= \begin{bmatrix}
1.072 + j0.22 \\
1.0 \\
\end{bmatrix}
\]
Results

- The below graph shows the results for four seconds of simulation, using Euler's with $\Delta t=0.01$ seconds

PowerWorld case is **B2_GENROU_2GEN_EULER**
Results for Longer Time

- Simulating out 10 seconds indicates an unstable solution, both using Euler's and RK2 with $\Delta t=0.005$, so it is really unstable!

Euler's with $\Delta t=0.01$

RK2 with $\Delta t=0.005$
Adding More Models

• In this situation the case is unstable because we have not modeled exciters

• To each generator add an EXST1 with $T_R = 0$, $T_C = T_B = 0$, $K_f = 0$, $K_A = 100$, $T_A = 0.1$

  This just adds one differential equation per generator

  \[
  \frac{dE_{FD}}{dt} = \frac{1}{T_A} \left( K_A \left( V_{REF} - |V_t| \right) - E_{FD} \right)
  \]
Two Bus, Two Gen With Exciters

- Below are the initial values for this case from PowerWorld

Because of the zero values the other differential equations for the exciters are included but treated as ignored

Case is B2_GENROU_2GEN_EXCITER
Viewing the States

- PowerWorld allows one to single-step through a solution, showing the $f(x)$ and the $K_1$ values
  - This is mostly used for education or model debugging

Derivatives shown are evaluated at the end of the time step
Two Bus Results with Exciters

- Below graph shows the angles with $\Delta t=0.01$ and a fault clearing at $t=0.05$ using Euler's
  - With the addition of the exciters case is now stable