ECEN 615
Methods of Electric Power Systems Analysis

Lecture 17: Sensitivity Analysis, Least Squares

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Announcements

• Read Chapter 9
  • We’ll just briefly cover state estimation since it is covered by ECEN 614, but will use it as an example for least squares and QR factorization

• Homework 4 is due on Thursday Nov 1
LODFs Evaluation Revisited

- We simulate the impact of the outage of line $k$ by adding the basic transaction $w_k = \{i', j', \Delta t_k\}$ and selecting $\Delta t_k$ in such a way that the flows on the dashed lines become exactly zero.

The $\Delta t_k$ is zeroing out the flow on the dashed lines; if we simulated in power flow the flow on the line itself would be quite high.
LODFs Evaluation Revisited Five Bus Example

Line 1
-52 MW
Line 2
26 MW
Line 3
63 MW
Line 4
-128 MW
Line 5
37 MW
Line 6
-0 MW

slack
1.050 pu
1.040 pu
1.042 pu
1.042 pu
1.044 pu

One
Two

A
MVA
A
MVA
A
MVA
A
MVA

200 MW
238 MW
1.050 pu

1.040 pu
1.042 pu

280 MW
100 MW

128 MW
0 MW
2a
2a

-100 MW
100 MW

86%

-100 MW
100 MW

Four
Five

3a
3a
LODFs Evaluation Revisited Five Bus Example

Line 1: 1.050 pu, -179 MW, 106 MW, -100 MW

Line 2: 1.040 pu, 238 MW, 100 MW

Line 3: 1.050 pu, 106 MW

Line 4: 1.028 pu, 111 MW, 1 MW

Line 5: 1.030 pu, 100 MW

Line 6: 1.030 pu, 100 MW

One: 200 MW, 1.050 pu

Two: 280 MW, 1.040 pu, 100 MW

Three: 118 MW, 1.017 pu

Four: 450 MW, 120% MVA

Five: 450 MW, 303% MVA

2a: -1 MW

3a: -1 MW
Multiple Line LODFs

- LODFs can also be used to represent multiple device contingencies, but it is usually more involved than just adding the effects of the single device LODFs.
- Assume a simultaneous outage of lines $k_1$ and $k_2$.
- Now setup two transactions, $w_{k1}$ (with value $\Delta t_{k1}$) and $w_{k2}$ (with value $\Delta t_{k2}$) so

\[
\begin{align*}
    f_{k1} + \Delta f_{k1} + \Delta f_{k2} - \Delta t_{k1} &= 0 \\
    f_{k2} + \Delta f_{k1} + \Delta f_{k2} - \Delta t_{k2} &= 0 \\
    f_{k1} + \varphi^{(w_{k1})} \Delta t_{k1} + \varphi^{(w_{k2})} \Delta t_{k2} - \Delta t_{k1} &= 0 \\
    f_{k2} + \varphi^{(w_{k1})} \Delta t_{k1} + \varphi^{(w_{k2})} \Delta t_{k1} - \Delta t_{k2} &= 0
\end{align*}
\]
Multiple Line LODFs

• Hence we can calculate the simultaneous impact of multiple outages; details for the derivation are given in C. Davis, T. J. Overbye, "Linear Analysis of Multiple Outage Interaction," Proc. 42nd HICSS, 2009

• Equation for the change in flow on line \( \ell \) for the outage of lines \( k_1 \) and \( k_2 \) is

\[
\Delta f_\ell = \begin{bmatrix} d_{\ell}^{k_1} & d_{\ell}^{k_2} \end{bmatrix} \left[ \begin{bmatrix} 1 & -d_{k_1}^{k_2} \\ -d_{k_2}^{k_1} & 1 \end{bmatrix} \right]^{-1} \begin{bmatrix} f_{k_1} \\ f_{k_2} \end{bmatrix}
\]
Multiple Line LODFs

- Example: Five bus case, outage of lines 2 and 5 to flow on line 4.

\[
\Delta f_\ell = \begin{bmatrix} d_{\ell}^{k_1} & d_{\ell}^{k_2} \end{bmatrix} \begin{bmatrix} 1 & -d_{k_2}^{k_1} \\ -d_{k_2}^{k_1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_{k_1} \\ f_{k_2} \end{bmatrix}
\]

\[
\Delta f_\ell = \begin{bmatrix} 0.4 & 0.25 \end{bmatrix} \begin{bmatrix} 1 & -0.75 \\ -0.6 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.336 \\ -0.331 \end{bmatrix} = 0.005
\]
Multiple Line LODFs

Flow goes from 117.5 to 118.0
Line Closure Distribution Factors (LCDFs)

base case

line k addition case

\[
LCDF_{\ell,k}^k = \frac{\Delta f_{\ell}}{f_k} = \text{LCDF}_{\ell,k}
\]
The line closure distribution factor (LCDF), \( LCDF_{\ell,k} \), for the closure of line \( k \) (or its addition if it does not already exist) is the portion of the line active power flow on line \( k \) that is distributed to line \( \ell \) due to the closure of line \( k \).

Since line \( k \) is currently open, the obvious question is, "what flow on line \( k \)?"

Answer (in a dc power flow sense) is the flow that will occur when the line is closed (which we do not know).
LCDF Evaluation

- We simulate the impact of the closure of line $k$ by imposing the additional basic transaction

$$w_k = \{i', j', \Delta t_k \}$$

on the base case network and we select $\Delta t_k$ so that

$$\Delta t_k = -f_k$$
LCDF Evaluation

- For the other parts of the network, the impacts of the addition of line $k$ are the same as the impacts of adding the basic transaction $w_k$

\[
\Delta f_{\ell} = \phi^{(w_k)}_{\ell} \Delta t_k = -\phi^{(w_k)}_{\ell} f_k
\]

- Therefore, the definition is

\[
LCDF_{\ell,k} = \frac{\Delta f_{\ell}}{f_k} = -\phi^{(w_k)}_{\ell}
\]

- The post-closure flow $f_k$ is determined (in a dc power flow sense) as the flow that would occur from the angle difference divided by $(1 + \phi^{(w_k)}_k)$
Outage Transfer Distribution Factor

- The outage transfer distribution factor (OTDF) is defined as the PTDF with the line k outaged.
- The OTDF applies only to the post-contingency configuration of the system since its evaluation explicitly considers the line k outage.

\[ \left( \phi_{(w)}^{(l)} \right)^k \]

- This is a quite important value since power system operation is usually contingency constrained.
Outage Transfer Distribution Factor (OTDF)

\[ f_\ell + \Delta f_\ell \]

\[ \left( \varphi^{(w)}_\ell \right)^k \triangleq \left. \frac{\Delta f_\ell}{\Delta t} \right|_{k \text{ outaged}} \]
OTDF Evaluation

\[ \Delta f_{\ell} = \Delta f^{(1)}_{\ell} \]  

\[ + \Delta f^{(2)}_{\ell} \]
OTDF Evaluation

- Since \( \Delta f^{(1)}_\ell = \varphi^{(w)}_\ell \Delta t \)

and \( \Delta f_k = \varphi^{(w)}_k \Delta t \)

then \( \Delta f^{(2)}_\ell = d^k \Delta f_k = d^k \varphi^{(w)}_k \Delta t \)

so that

\[
\Delta f_\ell = \Delta f^{(1)}_\ell + \Delta f^{(2)}_\ell = \left[ \varphi^{(w)}_\ell + d^k \varphi^{(w)}_k \right] \Delta t
\]

\[
\left( \varphi^{(w)}_\ell \right)^k = \varphi^{(w)}_\ell + d^k \varphi^{(w)}_k
\]
Five Bus Example

- Say we would like to know the PTDF on line 1 for a transaction between buses 2 and 3 with line 2 out.
Five Bus Example

- Hence we want to calculate these values without having to explicitly outage line 2

Hence the value we are looking for is 0.2 (20%)
Five Bus Example

- Evaluating: the PTDF for the bus 2 to 3 transaction on line 1 is 0.2727; it is 0.1818 on line 2 (from buses 1 to 3); the LODF is on line 1 for the outage of line 2 is -0.4
- Hence \( \left( \varphi_{(w)}^{(k)} \right) \) = \( \varphi_{(w)}^{(k)} + d_{(k)}^{(w)} \)

\[0.2727 + (-0.4) \times (0.1818) = 0.200\]

- For line 4 (buses 2 to 3) the value is

\[0.7273 + (0.4) \times (0.1818) = 0.800\]
August 14, 2003 OTDF Example

- Flowgate 2264 monitored the flow on Star-Juniper 345 kV line for contingent loss of Hanna-Juniper 345 kV normally the LODF for this flowgate is 0.361
  - flowgate had a limit of 1080 MW
  - at 15:05 EDT the flow as 517 MW on Star-Juniper, 1004 MW on Hanna-Juniper, giving a flowgate value of
    \[520 + 0.361 \times 1007 = 884\ (82\%)
  - Chamberlin-Harding 345 opened at 15:05, but was missed
  - At 15:06 EDT (after loss of Chamberlin-Harding 345) #2265 had an incorrect value because its LODF was not updated.
    - Value should be \[633 + 0.463 \times 1174 = 1176\ (109\%)
    - Value was \[633 + 0.361 \times 1174 = 1057\ (98\%)

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UTC Revisited

- We can now revisit the uncommitted transfer capability (UTC) calculation using PTDFs and LODFs
- Recall trying to determine maximum transfer between two areas (or buses in our example)
- For base case maximums are quickly determined with PTDFs

\[
 u^{(o)}_{m,n} = \min_{\phi^{(w)}_{\ell} > 0} \left\{ \frac{f_{\ell}^{\max} - f_{\ell}^{(o)}}{\phi^{(w)}_{\ell}} \right\}
\]

Note we are ignoring zero (or small) PTDFs; would also need to consider flow reversal
UTC Revisited

- For the contingencies we use

\[ u_{m,n}^{(1)} = \min \left\{ \frac{f_{\ell}^{\text{max}} - f_{\ell}^{(0)} - d_{k}^{\ell} f_{k}^{(0)}}{(\phi_{w}^{(w)})^{k}} \right\} \]

- Then as before \( u_{m,n} = \min \{ u_{m,n}^{(0)}, u_{m,n}^{(1)} \} \)

We would need to check all contingencies! Also, this is just a linear estimate and is not considering voltage violations.
Five Bus Example

\[ w = \{2, 3, \Delta t\} \quad f^{(0)} = [42, 34, 67, 118, 33, 100]^T \]

\[ f^{max} = [150, 400, 150, 150, 150, 1,000]^T \]
Therefore, for the base case

\[ u_{2,2}^{(0)} = \min_{\phi_{\ell}^{(w)}>0} \left\{ \frac{f_{\ell}^{\max} - f_{\ell}^{(0)}}{\phi_{\ell}^{(w)}} \right\} \]

\[
= \min \left\{ \frac{150 - 42}{0.2727}, \frac{400 - 34}{0.1818}, \frac{150 - 67}{0.0909}, \frac{150 - 118}{0.7273}, \frac{150 - 33}{0.0909} \right\}
\]

\[
= 44.0
\]
Five Bus Example

For the contingency case corresponding to the outage of the line 2

\[ u_{2,3}^{(1)} = \min \left\{ \frac{f_{\ell}^{\text{max}} - f_{\ell}^{(0)} - d_{\ell}^{2}f_{2}^{(0)}}{(\varphi_{\ell}^{(w)})^2} \right\} > 0 \]

The limiting value is line 4

\[ \frac{f_{\ell}^{\text{max}} - f_{\ell}^{(0)} - d_{\ell}^{2}f_{2}^{(0)}}{(\varphi_{\ell}^{(w)})^2} = \frac{150 - 118 - 0.4 \times 34}{0.8} = 26 \]

Hence the UTC is limited by the contingency to 23.0
Additional Comments

• Distribution factors are defined as small signal sensitivities, but in practice, they are also used for simulating large signal cases.

• Distribution factors are widely used in the operation of the electricity markets where the rapid evaluation of the impacts of each transaction on the line flows is required.

• Applications to actual systems show that the distribution factors provide satisfactory results in terms of accuracy.

• For multiple applications that require fast turn around time, distribution factors are used very widely, particularly, in the market environment.

• They do not work well with reactive power!
Least Squares

- So far we have considered the solution of $Ax = b$ in which $A$ is a square matrix; as long as $A$ is nonsingular there is a single solution
  - That is, we have the same number of equations ($m$) as unknowns ($n$)
- Many problems are overdetermined in which there are more equations than unknowns ($m > n$)
  - Overdetermined systems are usually inconsistent, in which no value of $x$ exactly solves all the equations
- Underdetermined systems have more unknowns than equations ($m < n$); they never have a unique solution but are usually consistent