Announcements

- Read Chapter 6
- Homework 1 is due today
- Homework 2 is due on Sept 27
US Balancing Authorities

U.S. electric power regions

Interconnections
- Eastern
- ERCOT
- Western

Circles represent the 66 balancing authorities
Area Control Error (ACE)

- The area control error is the difference between the actual flow out of an area, and the scheduled flow.
  - ACE also includes a frequency component that we will probably consider later in the semester.
- Ideally the ACE should always be zero.
- Because the load is constantly changing, each utility (or ISO) must constantly change its generation to “chase” the ACE.
- ACE was originally computed by utilities; increasingly it is computed by larger organizations such as ISOs.
Most utilities (ISOs) use automatic generation control (AGC) to automatically change their generation to keep their ACE close to zero.

Usually the control center calculates ACE based upon tie-line flows; then the AGC module sends control signals out to the generators every couple seconds.
Net tie flow is close to zero

Generation is automatically changed to match change in load

Three Bus Case on AGC
Generator Costs

- There are many fixed and variable costs associated with power system operation
- The major variable cost is associated with generation.
- Cost to generate a MWh can vary widely
- For some types of units (such as hydro and nuclear) it is difficult to quantify
- More others such as wind and solar the marginal cost of energy is essentially zero (actually negative for wind!)
- For thermal units it is straightforward to determine
- Many markets have moved from cost-based to price-based generator costs
Economic Dispatch

- Economic dispatch (ED) determines the least cost dispatch of generation for an area.
- For a lossless system, the ED occurs when all the generators have equal marginal costs.

\[ IC_1(P_{G,1}) = IC_2(P_{G,2}) = \ldots = IC_m(P_{G,m}) \]
Power Transactions

• Power transactions are contracts between areas to do power transactions.
• Contracts can be for any amount of time at any price for any amount of power.
• Scheduled power transactions are implemented by modifying the area ACE:

\[ \text{ACE} = P_{\text{actual,tie-flow}} - P_{\text{sched}} \]
100 MW Transaction

Scheduled 100 MW Transaction from Left to Right

Net tie-line flow is now 100 MW
Security Constrained ED

- Transmission constraints often limit system economic operation.
- Such limits required a constrained dispatch in order to maintain system security.
- In the three bus case the generation at bus 3 must be constrained to avoid overloading the line from bus 2 to bus 3.
Dispatch is no longer optimal due to need to keep the line from bus 2 to bus 3 from overloading.
Multi-Area Operation

- If areas have direct interconnections then they may directly transact, up to the capacity of their tie-lines.
- Actual power flows through the entire network according to the impedance of the transmission lines.
- Flow through other areas is known as “parallel path” or “loop flow.”
Seven Bus Case: One-line

System has three areas

Area Top has five buses

Area Left has one bus

Area Right has one bus

PowerWorld Case: B7Flat
Seven Bus Case: Area View

System has 40 MW of “Loop Flow”

Actual flow between areas

Scheduled flow

Loop flow can result in higher losses
Seven Bus - Loop Flow?

Transaction has actually decreased the loop flow

Note that Top’s losses have increased from 7.09 MW to 9.44 MW

100 MW Transaction between Left and Right
Power Transfer Distribution Factors (PTDFs)

• PTDFs are used to show how a particular transaction will affect the system

• The power transfers through the system according to the impedances of the lines, without respect to ownership

• All transmission players in network could potentially be impacted (to a greater or lesser extent)

• Later in the semester we’ll consider techniques for calculating PTDFs
PTDF Example: Nine Bus System, Actual Flows

PowerWorld Case: B9
PTDF Example: Nine Bus System, Transfer from A to I

Values now tell percentage of flow that will go on line
PTDF Example: Nine Bus System, Transfer From G to F

Values now tell percentage of flow that will go on line
Wisconsin to TVA Line PTDF Contour

Contours show lines that would carry at least 2% of a power transfer from Wisconsin to TVA
NERC Flowgates

- One common term is a “flowgate,” which is a mathematical construct to measure the MW flow on one or more elements in the bulk transmission system.
  - Sometimes they include the impact of contingencies, something we will consider later in the semester.
- A simple flowgate would be the MW flow through a single transmission line or transformer.
NERC TLRs

- In the North American transmission loading relief procedures (TLRs) are used to mitigate the overloads on the bulk transmission system
  - Called TLR in the East, WECC Unscheduled Flow Mitigation or Congestion Management Procedures (ERCOT)
- In the Eastern Interconnect TLRs consider the PTDFs associated with transactions on flowgates if there is a flowgate violation
During summer of 1998 congestion on just two elements pushed Midwest spot market prices up by a factor of 200: from $20/MWh to $7500/MWh!
Pricing Electricity

- Cost to supply electricity to bus is called the locational marginal price (LMP)
- Presently some electric markets post LMPs on the web
- In an ideal electricity market with no transmission limitations the LMPs are equal
- Transmission constraints can segment a market, resulting in differing LMP
- Determination of LMPs requires the solution on an Optimal Power Flow (OPF), which will be covered later in the semester
Three Bus LMPs – Constraints Ignored

Gen 2’s cost is $12 per MWh

Gen 1’s cost is $10 per MWh

Line from Bus 1 to Bus 3 is over-loaded; all buses have same marginal cost

PowerWorld Case: B3LP
Line from 1 to 3 is no longer overloaded, but now the marginal cost of electricity at bus 3 is $14 / MWh
Five minute LMPs are posted online for the MISO footprint

Source: https://www.misoenergy.org/markets-and-operations/real-time-displays/
Advanced Power Flow

• Next slides cover some more advanced power flow topics that need to be considered in many commercial power flow studies

• An important consideration in the power flow is the assumed time scale of the response, and the assumed model of operator actions
  • Planning power flow studies usually assume automatic modeling of operator actions and a longer time frame of response (controls have time to reach steady-state)
    • For example, who is actually doing the volt/var control
  • In real-time applications operator actions are usually not automated and controls may be more limited in time
Power Flow Optimal Multiplier

- Key idea is once NR method has selected a direction, we can analytically determine the distance to move in that direction to minimize the norm of the mismatch
  - Goal is to help with stressed power systems
Power Flow with Optimal Multiplier

- Consider an n bus power system with $f(x) = S$ where $S$ is the vector of the constant real and reactive power load minus generation at all buses except the slack, $x$ is the vector of the bus voltages in rectangular coordinates: $V_i = e_i + jf_i$, and $f$ is the function of the power balance constraints

$$f_{pi} = \sum_{j=1}^{n} \left( e_i \left( e_j G_{ij} - f_j B_{ij} \right) + f_i \left( f_j G_{ij} + e_j B_{ij} \right) \right)$$

$$f_{qi} = \sum_{j=1}^{n} \left( f_i \left( e_j G_{ij} - f_j B_{ij} \right) - e_i \left( f_j G_{ij} + e_j B_{ij} \right) \right)$$

$G + jB$ is the bus admittance matrix
Power Flow with Optimal Multiplier

- With a standard NR approach we would get

\[ x^{k+1} = x^k + \Delta x^k \]

\[ \Delta x^k = -J(x^k)^{-1}(f(x^k) - S) \]

- If we are close enough to the solution the iteration converges quickly, but if the system is heavily loaded it can diverge
Power Flow with Optimal Multiplier

- Optimal multiplier approach modifies the iteration as

\[ x^{k+1} = x^k + \mu \Delta x^k \]

\[ \Delta x^k = -J(x^k)^{-1} \left( f(x^k) - S \right) \]

- Scalar \( \mu \) is chosen to minimize the norm of the mismatch \( F \) in direction \( \Delta x \)

\[ F(x^{k+1}) = \frac{1}{2} \left[ f(x^k + \mu \Delta x^k) - S \right]^T \left[ f(x^k + \mu \Delta x^k) - S \right] \]

- Paper by Iwamoto, Y. Tamura from 1981 shows \( \mu \) can be computed analytically with little additional calculation when rectangular voltages are used
Power Flow with Optimal Multiplier

- Determination of $\mu$ involves solving a cubic equation, which gives either three real solutions, or one real and two imaginary solutions.
- 1989 PICA paper by Iba ("A Method for Finding a Pair of Multiple Load Flow Solutions in Bulk Power Systems") showed that NR tends to converge along line joining the high and a low voltage solution.

However, there are some model restrictions.
Quasi-Newton Power Flow Methods

- First we consider some modified versions of the Newton power flow (NPF)
- Since most of the computation in the NPF is associated with building and factoring the Jacobian matrix, $J$, the focus is on trying to reduce this computation
- In a pure NPF $J$ is build and factored each iteration
- Over the years pretty much every variation of the NPF has been tried; here we just touch on the most common
- Whether a method is effective can be application dependent
  - For example, in contingency analysis we are usually just resolving a solved case with an often small perturbation
Quasi-Newton Power Flow Methods

• The simplest modification of the NPF results when $\mathbf{J}$ is kept constant for a number of iterations, say $k$ iterations
  • Sometimes known as the Dishonest Newton
• The approach balances increased speed per iteration, with potentially more iterations to perform
• There is also an increased possibility for divergence
• Since the mismatch equations are not modified, if it converges it should converge to the same solution as the NPF
• These methods are not commonly used, except in very short duration, sequential power flows with small mismatches
Dishonest N-R Example

\[
x^{(v+1)} = x^{(v)} - \left[ \frac{1}{2x^{(0)}} \right] ((x^{(v)})^2 - 2)
\]

Guess \(x^{(0)} = 1\). Iteratively solving we get

<table>
<thead>
<tr>
<th>(v)</th>
<th>(x^{(v)}) (honest)</th>
<th>(x^{(v)}) (dishonest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>1.41667</td>
<td>1.375</td>
</tr>
<tr>
<td>3</td>
<td>1.41422</td>
<td>1.429</td>
</tr>
<tr>
<td>4</td>
<td>1.41422</td>
<td>1.408</td>
</tr>
</tbody>
</table>

We pay a price in increased iterations, but with decreased computation per iteration; that price is too high in this example.
NPF (Honest) Region of for Two Bus Example Convergence

Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution.

Maximum of 15 iterations
Two Bus Dishonest ROC

In this case being honest pays! At least with respect to the region of convergence (ROC)
A second modification is to modify the step size in the direction given by the NPF

This is one we’ve already considered with the optimal multiplier approach

\[
\Delta x^k = -J(x^k)^{-1}(f(x^k) - S)
\]

\[
x^{k+1} = x^k + \mu \Delta x^k
\]

The generalized approach is to solve what is known as the line search (i.e., a one-dimensional optimization) to determine \( \mu \)
The Single Dimensional $\psi(\lambda)$

$\psi(0) = F(x_i)$

$\psi(\lambda) = F[x_i + \lambda h(x_i)]$

$\lambda^*$
Line Search

- We need a cost function, which is usually the Euclidean norm of the mismatch vector.
- The line search is a general optimization problem for which there are many potential solution approaches.
  - Determines a local optimum within some search boundaries.
  - Approaches depend on whether there is gradient information available.
- Aside from the optimal multiplier approach, which can be quite helpful with little additional computation, the convergence gain from determining the “optimal” \( \mu \) is usually more than offset by the line search computation.
Decoupled Power Flow

- Rather than not updating the Jacobian, the decoupled power flow takes advantage of characteristics of the power grid in order to decouple the real and reactive power balance equations
  - There is a strong coupling between real power and voltage angle, and reactive power and voltage magnitude
  - There is a much weaker coupling between real power and voltage angle, and reactive power and voltage angle
Decoupled Power Flow Formulation

General form of the power flow problem

\[
- \begin{bmatrix}
\frac{\partial P^{(v)}}{\partial \theta} & \frac{\partial P^{(v)}}{\partial |V|} \\
\frac{\partial Q^{(v)}}{\partial \theta} & \frac{\partial Q^{(v)}}{\partial |V|}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta^{(v)} \\
\Delta |V|^{(v)}
\end{bmatrix}
= \begin{bmatrix}
\Delta P(x^{(v)}) \\
\Delta Q(x^{(v)})
\end{bmatrix} = f(x^{(v)})
\]

where

\[
\Delta P(x^{(v)}) = \begin{bmatrix}
P_2(x^{(v)}) + P_{D2} - P_{G2} \\
\vdots \\
P_n(x^{(v)}) + P_{Dn} - P_{Gn}
\end{bmatrix}
\]
Decoupling Approximation

Usually the off-diagonal matrices, \( \frac{\partial P}{\partial \theta} \) and \( \frac{\partial Q}{\partial \theta} \) are small. Therefore we approximate them as zero:

\[
- \begin{bmatrix}
    \frac{\partial P^{(v)}}{\partial \theta} & 0 \\
    0 & \frac{\partial Q^{(v)}}{\partial |V^{(v)}|}
\end{bmatrix}
\begin{bmatrix}
    \Delta \theta^{(v)} \\
    \Delta |V^{(v)}|
\end{bmatrix}
= \begin{bmatrix}
    \Delta P(x^{(v)}) \\
    \Delta Q(x^{(v)})
\end{bmatrix}
= f(x^{(v)})
\]

Then the problem can be decoupled

\[
\Delta \theta^{(v)} = -\left[ \frac{\partial P^{(v)}}{\partial \theta} \right]^{-1} \Delta P(x^{(v)}) \Delta |V^{(v)}| = -\left[ \frac{\partial Q^{(v)}}{\partial |V^{(v)}|} \right]^{-1} \Delta Q(x^{(v)})
\]
Off-diagonal Jacobian Terms

Justification for Jacobian approximations:

1. Usually \( r \ll x \), therefore \( |G_{ij}| \ll |B_{ij}| \)

2. Usually \( \theta_{ij} \) is small so \( \sin \theta_{ij} \approx 0 \)

Therefore

\[
\frac{\partial P_i}{\partial |V_j|} = |V_i| \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) \approx 0
\]

\[
\frac{\partial Q_i}{\partial \theta_j} = -|V_i| |V_j| \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) \approx 0
\]

By assuming \( \frac{1}{2} \) the elements are zero, we only have to do \( \frac{1}{2} \) the computations.
Decoupled N-R Region of Convergence

The high solution ROC is actually larger than with the standard NPF. Obviously this is not a good a way to get the low solution.