HW 6

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A 1) $P_L = 4 + \frac{16}{100} = 4.16$ pu

$B = -10$

$$-\frac{P_L^2}{B} + Q_L + \frac{B}{4} = 0$$

$\Rightarrow$ for $P = 4.16$ pu; $Q = 0.769$ pu

Analytically solving for V and θ by substitution :

$$B^2V^4 - (2BQ_L + B^2)V^2 + (P_L^2 + Q_L^2) = 0$$

If $x = V^2$, then $B^2x^2 - (2BQ_L + B^2)x + (P_L^2 + Q_L^2) = 0$

$$\Rightarrow x = \frac{(2BQ_L + B^2) \pm \sqrt{((2BQ_L + B^2)^2 - 4(B^2)(P_L^2 + Q_L^2)}}{2B^2}$$

thus $V = \sqrt{x}$

~~2 (-10)~~ On solving with $P_L = 4.16$, $Q_L = 0.769$ & $B = -10$,

we get. $V_1 \approx V_2 \approx 0.656$ pu & $\theta_1 \approx \theta_2 \approx -39.376°$

$$J(\theta, V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$

$$= \begin{bmatrix} 5.07 & -6.344 \\ -4.162 & 5.27 \end{bmatrix}$$

solving for left eigen vector ($\lambda = 0$) :

$$w = \begin{bmatrix} -0.6803 \\ 0.7329 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P_L^{new} \\ Q_L^{new} \end{bmatrix} = \begin{bmatrix} P_L^{old} \\ Q_L^{old} \end{bmatrix} + \beta \cdot w$$

take ~~where~~ $\beta = 0.1$

$$\therefore \begin{bmatrix} P_L^{new} \\ Q_L^{new} \end{bmatrix} = \begin{bmatrix} 4.092 \\ 0.69571 \end{bmatrix}$$

Using this, we get a new eq^n :

$$100x^2 - ((2)(-10)(0.696) + 100)x + (4.092^2 + 0.696^2) = 0$$

we get $x = 0.544, \ 0.3166$

$\Rightarrow V_1 = \sqrt{0.544}$, $V_2 = \sqrt{0.3166}$

$= 0.7375$ pu , $= 0.5626$ pu

$\Rightarrow \theta_1 = \sin^{-1}\left(\frac{4.092}{-10 \times 0.7375}\right)$; $\theta_2 = \sin^{-1}\left(\frac{4.092}{-10 \times 0.5626}\right)$

$= -33.700°$ $= -46.664°$

$= -0.588$ radians $= -0.814$ radians

Solving for right-eigen vector ($\lambda = 0$) :

$$V = \begin{bmatrix} 0.771 \\ -0.6368 \end{bmatrix}$$

$$diff = \begin{bmatrix} \theta_{2\,new} \\ V_{2\,new} \end{bmatrix} - \begin{bmatrix} \theta_{1\,new} \\ V_{1\,new} \end{bmatrix} = \begin{bmatrix} -0.226 \\ -0.1749 \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{diff \cdot v}{|diff| \cdot |v|}\right) = 0.56°$$

Since $\theta \approx 0°$, indicates v & diff are in the same direction.

(A.3) $L(x, \lambda) = (x_1-3)^2 + (x_2-4)^2 + \lambda(x_1 + x_2 - 5)$

λ is the Lagrange Multiplier

$$\nabla L = \begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \\ \frac{\partial L}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} 2(x_1-3)+\lambda \\ 2(x_2-4)+\lambda \\ x_1+x_2-5 \end{bmatrix}$$

$$\Rightarrow H = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

let $z^{0} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix}$

$$z^{k+1} = z^k - H^{-1}[\nabla L(z^k)]$$

$$\Rightarrow z^1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ -6 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$\Rightarrow x_1 = 2$; $x_2 = 3$; $\lambda = 2$

4.4

x_1	x_2	x_3	x_4	
2	2	1	0	10
4	8	0	1	12
-68	-116	0	0	0

$10/2 = 5$

$12/8 = 1.5 \leftarrow$ least +ve

pivot element

most -ve

(1) $\frac{R_2}{8} \rightarrow R_2$

(2) $2R_2 - R_1 \rightarrow R_1$ & $116 R_2 + R_3 \rightarrow R_3$

$\Rightarrow$

x_1	x_2	x_3	x_4	
-1	0	-1	1/4	-7
1/2	1	0	1/8	3/2
-10	0	0	29/2	174

$-7/-1 = 7$

$(3/2)/(1/2) = 3$

-ve

(1) $2R_2 \rightarrow R_2$

(2) $\begin{cases} R_2 + R_1 \rightarrow R_1 \\ 10R_2 + R_3 \rightarrow R_3 \end{cases}$

x_1	x_2	x_3	x_4	
0	2	-1	1/2	-4
1	2	0	1/4	3
0	20	0	17	204

max

→ no -ve value in last column.

⇒ optimal //

Since x_1 & x_3 only have 1 non-zero value, they are basic variables.

Whereas, x_2 & x_4 are non-basic.

$\therefore$ $x_1 = 3$; $x_2 = 0$; $x_3 = 4$; $x_4 = 0$ //

&

maximum value of function = 204 //