Announcements

• Read Chapter 6 from the book
  • The bookformulates the power flow using the polar form for the $Y_{bus}$ elements

• Homework 2 is due on Thursday September 17
Three Bus PowerWorld Simulator Case

PowerWorld Case Name: B3Slow

Load with green arrows indicating amount of MW flow

Used to control output of generator

Direction of green arrow is used to indicate direction of real power (MW) flow; the blue arrows show the reactive power

Area Name: Home
ACE: -15.5 MW
MW Load: 316.2 MW
MW Gen: 301.0 MW
MW Losses: 0.28 MW

Bus 2

Bus 1

Bus 3

Home Area

Other Area

Scheduled Transactions

0.0 MW
Off AGC

151.0 MW
105.4 Mvar
AGC OFF
AVR ON

210.8 MW
105.4 Mvar

25.4 MW
5.3 Mvar

1.00 pu

1.00 pu

10.1 MW
3.1 Mvar

115.4 MW
-1.9 Mvar

100 MW

105.4 MW
52.7 Mvar

150.0 MW
39.7 Mvar
AVR ON AGC ON

ACE: -15.5 MW

121.3 MW
39.7 Mvar

0.0 MW

Off AGC
Basic Power Control

- Opening a circuit breaker causes the power flow to instantaneously (nearly) change.
- No other way to directly control power flow in a transmission line.
- By changing generation we can indirectly change this flow.
- Power flow in transmission line is limited by heating considerations
- Losses ($I^2 R$) can heat up the line, causing it to sag.
Transmission Line Limits

- Power flow in transmission line is limited by heating considerations.
- Losses ($I^2 R$) can heat up the line, causing it to sag.
- Each line has a limit; many utilities use winter/summer limits.
Area Name: Home
ACE: -263.1 MW
MW Load: 559.2 MW
MW Gen: 301.0 MW
MW Losses: 4.91 MW

Scheduled Transactions

Bus 2
372.8 MW
186.4 Mvar

151.0 MW AGC OFF
245.0 Mvar

AGC OFF
AVR ON

Bus 3
150.0 MW
107.9 Mvar

AGC OFF
AVR ON

Bus 1
363.0 MW
-52.3 Mvar

100.0 MW

Off AGC
Interconnected Operation
Balancing Authority (BA) Areas

• North American Eastern and Western grids are divided into balancing authority areas (BA)
  – Often just called an area

• Transmission lines that join two areas are known as tie-lines.

• The net power out of an area is the sum of the flow on its tie-lines.

• The flow out of an area is equal to

  total gen - total load - total losses = tie-flow
US Balancing Authorities

U.S. electric power regions

Interconnections
- Eastern
- ERCOT
- Western

Circles represent the 66 balancing authorities
Area Control Error (ACE)

- The area control error is the difference between the actual flow out of an area, and the scheduled flow
  - ACE also includes a frequency component that we will probably consider later in the semester
- Ideally the ACE should always be zero
- Because the load is constantly changing, each utility (or ISO) must constantly change its generation to “chase” the ACE
- ACE was originally computed by utilities; increasingly it is computed by larger organizations such as ISOs
Automatic Generation Control

• Most utilities (ISOs) use automatic generation control (AGC) to automatically change their generation to keep their ACE close to zero.

• Usually the control center calculates ACE based upon tie-line flows; then the AGC module sends control signals out to the generators every couple seconds.
Three Bus Case on AGC

Area Name: Home
ACE: -0.0 MW
MW Load: 330.2 MW
MW Gen: 330.6 MW
MW Losses: 0.40 MW

Generation is automatically changed to match change in load

Net tie flow is close to zero
Generator Costs

- There are many fixed and variable costs associated with power system operation.
- The major variable cost is associated with generation.
- Cost to generate a MWh can vary widely.
- For some types of units (such as hydro and nuclear) it is difficult to quantify.
- More others such as wind and solar the marginal cost of energy is essentially zero (actually negative for wind!).
- For thermal units it is straightforward to determine.
- Many markets have moved from cost-based to price-based generator costs.
Economic Dispatch

- Economic dispatch (ED) determines the least cost dispatch of generation for an area.
- For a lossless system, the ED occurs when all the generators have equal marginal costs.

\[ IC_1(P_{G,1}) = IC_2(P_{G,2}) = \ldots = IC_m(P_{G,m}) \]
Power Transactions

- Power transactions are contracts between areas to do power transactions.
- Contracts can be for any amount of time at any price for any amount of power.
- Scheduled power transactions are implemented by modifying the area ACE:

\[ \text{ACE} = \text{P}_{\text{actual, tie-flow}} - \text{P}_{\text{sched}} \]
100 MW Transaction

Scheduled 100 MW transaction from the Home Area to the Other Area

Net tie-line flow is now 100 MW
Security Constrained ED

- Transmission constraints often limit system economic operation.
- Such limits required a constrained dispatch in order to maintain system security.
- In the three bus case the generation at bus 3 must be constrained to avoid overloading the line from bus 2 to bus 3.
Security Constrained Dispatch

Dispatch is no longer optimal due to need to keep the line from bus 2 to bus 3 from overloading.
Multi-Area Operation

• If areas have direct interconnections then they may directly transact, up to the capacity of their tie-lines.
• Actual power flows through the entire network according to the impedance of the transmission lines.
• Flow through other areas is known as “parallel path” or “loop flow.”
Seven Bus Case: One-line

System has three areas

Area Top has five buses

PowerWorld Case: B7Flat

Area Left has one bus

Area Right has one bus
Seven Bus Case: Area View

System has 40 MW of “Loop Flow”

Actual flow between areas

Scheduled flow

- Area Losses: 7.1 MW
- Scheduled flow: 40.2 MW
- Actual flow: 0.0 MW
- Area Losses: 0.3 MW
- Area Losses: 0.7 MW
Seven Bus - Loop Flow?

Transaction has actually decreased the loop flow.

Note that Top’s losses have increased from 7.1 MW to 9.6 MW.

100 MW Transaction between Left and Right.
Pricing Electricity

- Cost to supply electricity to bus is called the locational marginal price (LMP)
- Presently some electric markets post LMPs on the web
- In an ideal electricity market with no transmission limitations the LMPs are equal
- Transmission constraints can segment a market, resulting in differing LMP
- Determination of LMPs requires the solution on an Optimal Power Flow (OPF), which will be covered later in the semester
Three Bus LMPs – Constraints Ignored

- Gen 1’s cost is $10 per MWh
- Gen 2’s cost is $12 per MWh

Line from Bus 1 to Bus 3 is over-loaded; all buses have same marginal cost

PowerWorld Case: B3LP
Line from 1 to 3 is no longer overloaded, but now the marginal cost of electricity at bus 3 is $14 / MWh.
Five minute LMPs are posted online for the MISO footprint.

Source: https://www.misoenergy.org/markets-and-operations/real-time--market-data/real-time-displays/
Advanced Power Flow

• Next slides cover some more advanced power flow topics that need to be considered in many commercial power flow studies

• An important consideration in the power flow is the assumed time scale of the response, and the assumed model of operator actions
  
  – Planning power flow studies usually assume automatic modeling of operator actions and a longer time frame of response (controls have time to reach steady-state)
    
    • For example, who is actually doing the volt/var control

  – In real-time applications operator actions are usually not automated and controls may be more limited in time
Quasi-Newton Power Flow Methods

• First we consider some modified versions of the Newton power flow (NPF)
• Since most of the computation in the NPF is associated with building and factoring the Jacobian matrix, $J$, the focus is on trying to reduce this computation
• In a pure NPF $J$ is build and factored each iteration
• Over the years pretty much every variation of the NPF has been tried; here we just touch on the most common
• Whether a method is effective can be application dependent
  – For example, in contingency analysis we are usually just resolving a solved case with an often small perturbation
Quasi-Newton Power Flow Methods

• The simplest modification of the NPF results when $J$ is kept constant for a number of iterations, say $k$ iterations
  – Sometimes known as the Dishonest Newton
• The approach balances increased speed per iteration, with potentially more iterations to perform
• There is also an increased possibility for divergence
• Since the mismatch equations are not modified, if it converges it should converge to the same solution as the NPF
• These methods are not commonly used, except in very short duration, sequential power flows with small mismatches
**Dishonest N-R Example**

\[ x^{(v+1)} = x^{(v)} - \left( \frac{1}{2x^{(0)}} \right) \left( (x^{(v)})^2 - 2 \right) \]

Guess \( x^{(0)} = 1 \). Iteratively solving we get

\[
\begin{array}{cccc}
\text{v} & x^{(v)} \text{ (honest)} & x^{(v)} \text{ (dishonest)} \\
0 & 1 & 1 \\
1 & 1.5 & 1.5 \\
2 & 1.41667 & 1.375 \\
3 & 1.41422 & 1.429 \\
4 & 1.41422 & 1.408 \\
\end{array}
\]

We pay a price in increased iterations, but with decreased computation per iteration; that price is too high in this example.
Decoupled Power Flow

• Rather than not updating the Jacobian, the decoupled power flow takes advantage of characteristics of the power grid in order to decouple the real and reactive power balance equations
  – There is a strong coupling between real power and voltage angle, and reactive power and voltage magnitude
  – There is a much weaker coupling between real power and voltage angle, and reactive power and voltage angle

Decoupled Power Flow Formulation

General form of the power flow problem

\[ \begin{bmatrix} \frac{\partial P^{(v)}}{\partial \theta} & \frac{\partial P^{(v)}}{\partial |V|} \\ \frac{\partial Q^{(v)}}{\partial \theta} & \frac{\partial Q^{(v)}}{\partial |V|} \end{bmatrix} \begin{bmatrix} \Delta \theta^{(v)} \\ \Delta |V|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta P(x^{(v)}) \\ \Delta Q(x^{(v)}) \end{bmatrix} = f(x^{(v)}) \]

where

\[ \Delta P(x^{(v)}) = \begin{bmatrix} P_2(x^{(v)}) + P_{D2} - P_{G2} \\ \vdots \\ P_n(x^{(v)}) + P_{Dn} - P_{Gn} \end{bmatrix} \]
Decoupling Approximation

Usually the off-diagonal matrices, \( \frac{\partial P}{\partial |V|} \) and \( \frac{\partial Q}{\partial \theta} \) are small. Therefore we approximate them as zero:

\[
- \begin{bmatrix}
\frac{\partial P^{(v)}}{\partial \theta} & 0 \\
0 & \frac{\partial Q^{(v)}}{\partial |V|}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta^{(v)} \\
\Delta |V|^{(v)}
\end{bmatrix} =
\begin{bmatrix}
\Delta P(x^{(v)}) \\
\Delta Q(x^{(v)})
\end{bmatrix} = f(x^{(v)})
\]

Then the problem can be decoupled

\[
\Delta \theta^{(v)} = -\left[ \frac{\partial P^{(v)}}{\partial \theta} \right]^{-1} \Delta P(x^{(v)}) \Delta |V|^{(v)} = -\left[ \frac{\partial Q^{(v)}}{\partial |V|} \right]^{-1} \Delta Q(x^{(v)})
\]
Off-diagonal Jacobian Terms

Justification for Jacobian approximations:

1. Usually $r \ll x$, therefore $|G_{ij}| \ll |B_{ij}|$

2. Usually $\theta_{ij}$ is small so $\sin \theta_{ij} \approx 0$

Therefore

$$\frac{\partial P_i}{\partial |V_j|} = |V_i| \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) \approx 0$$

$$\frac{\partial Q_i}{\partial \theta_j} = -|V_i||V_j| \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) \approx 0$$

By assuming $\frac{1}{2}$ the elements are zero, we only have to do $\frac{1}{2}$ the computations
Decoupled N-R Region of Convergence

The high solution ROC is actually larger than with the standard NPF. Obviously this is not a good a way to get the low solution.
By continuing with our Jacobian approximations we can actually obtain a reasonable approximation that is independent of the voltage magnitudes/angles.

This means the Jacobian need only be built/inverted once per power flow solution.

This approach is known as the fast decoupled power flow (FDPF).
Fast Decoupled Power Flow, cont.

- FDPF uses the same mismatch equations as standard power flow (just scaled) so it should have same solution
- The FDPF is widely used, though usually only when we only need an approximate solution
- Modified versions also exist, such as D. Jajicic and A. Bose, “A Modification to the Fast Decoupled Power Flow for Networks with High R/X Ratios,” *IEEE Transactions on Power Sys.*, May 1988, pp. 743-746
FDPF Approximations

The FDPF makes the following approximations:

1. \( |G_{ij}| = 0 \)
2. \( |V_i| = 1 \)
3. \( \sin \theta_{ij} = 0 \quad \cos \theta_{ij} = 1 \)

To see the impact on the real power equations recall

\[
P_i = \sum_{k=1}^{n} V_i V_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) = P_{Gi} - P_{Di}
\]

Which can also be written as

\[
\frac{P_i}{V_i} = \sum_{k=1}^{n} V_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) = \frac{P_{Gi} - P_{Di}}{V_i}
\]
FDPF Approximations

• With the approximations for the diagonal term we get

\[ \frac{\partial P_i}{\partial \theta_i} \approx \sum_{k=1}^{n} B_{ik} = -B_{ii} \]

The for the off-diagonal terms \((k \neq i)\) with \(G=0\) and \(V=1\)

\[ \frac{\partial P_i}{\partial \theta_k} = -B_{ik} \cos \theta_{ik} \approx -B_{ik} \]

• Hence the Jacobian for the real equations can be approximated as \(-B\)
FPDF Approximations

- For the reactive power equations we also scale by $V_i$

$$Q_i = \sum_{k=1}^{n} V_i V_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) = Q_{Gi} - Q_{Di}$$

$$\frac{Q_i}{V_i} = \sum_{k=1}^{n} V_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) = \frac{Q_{Gi} - Q_{Di}}{V_i}$$

- For the Jacobian off-diagonals we get

$$\frac{\partial Q_i}{\partial V_k} = -B_{ik} \cos \theta_{ik} \approx -B_{ik}$$
FDPF Approximations

• And for the reactive power Jacobian diagonal we get
  \[ \frac{\partial Q_i}{\partial V_i} \approx -2B_{ii} - \sum_{k=1, k\neq i}^{n} B_{ik} = -B_{ii} \]

• As derived the real and reactive equations have a constant Jacobian equal to \(-B\)
  - Usually modifications are made to omit from the real power matrix elements that affect reactive flow (like shunts) and from the reactive power matrix elements that affect real power flow, like phase shifters
  - We’ll call the real power matrix \(B'\) and the reactive \(B''\)
FDPF Region of Convergence
FDPF Cautions

• The FDPF works well as long as the previous approximations hold for the entire system
• With the movement towards modeling larger systems, with more of the lower voltage portions of the system represented (for which r/x ratios are higher) it is quite common for the FDPF to get stuck because small portions of the system are ill-behaved
• The FDPF is commonly used to provide an initial guess of the solution or for contingency analysis
DC Power Flow

• The “DC” power flow makes the most severe approximations:
  – completely ignore reactive power, assume all the voltages are always 1.0 per unit, ignore line conductance

• This makes the power flow a linear set of equations, which can be solved directly

$$\theta = -B^{-1} P$$

• The term dc power flow actually dates from the time of the old network analyzers (going back into the 1930’s)

• Not to be confused with the inclusion of HVDC lines in the standard NPF

\[ P \] sign convention is generation is positive
I don’t think a classic dc power flow paper exists; a nice formulation is given in our book *Power Generation and Control* book by Wood, Wollenberg and Sheble

The August 2009 paper in IEEE Transactions on Power Systems, “DC Power Flow Revisited” (by Stott, Jardim and Alsac) provides good coverage

DC Power Flow Example

EXAMPLE 6.17

Determine the dc power flow solution for the five bus from Example 6.9.

SOLUTION  With bus 1 as the system slack, the $B$ matrix and $P$ vector for this system are

$$
B = \begin{bmatrix}
-30 & 0 & 10 & 20 \\
0 & -100 & 100 & 0 \\
10 & 100 & -150 & 40 \\
20 & 0 & 40 & -110
\end{bmatrix} \\
\quad P = \begin{bmatrix}
-8.0 \\
4.4 \\
0 \\
0
\end{bmatrix}
$$

$$
\delta = -B^{-1}P = \begin{bmatrix}
-0.3263 \\
0.0091 \\
-0.0349 \\
-0.0720
\end{bmatrix} \text{ radians = } \begin{bmatrix}
-18.70 \\
0.5214 \\
-2.000 \\
-4.125
\end{bmatrix} \text{ degrees}
$$
DC Power Flow in PowerWorld

- PowerWorld allows for easy switching between the dc and ac power flows (case Aggieland37)

To use the dc approach in PowerWorld select Tools, Solve, DC Power Flow

Notice there are no losses
Modeling Transformers with Off-Nominal Taps and Phase Shifts

• If transformers have a turns ratio that matches the ratio of the per unit voltages than transformers are modeled in a manner similar to transmission lines.

• However it is common for transformers to have a variable tap ratio; this is known as an “off-nominal” tap ratio
  – The off-nominal tap is $t$, initially we’ll consider it a real number
  – We’ll cover phase shifters shortly in which $t$ is complex
Transformer Representation

• The one–line diagram of a branch with a variable tap transformer

• The network representation of a branch with off–nominal turns ratio transformer is

\[ y_{km} = g_{km} + j b_{km} \]
Transformer Nodal Equations

• From the network representation

\[
\bar{I}_m = \bar{I}_{k'} = \bar{y}_{km} \left( \bar{E}_m - \bar{E}_{k'} \right) = \bar{y}_{km} \left( \bar{E}_m - \frac{\bar{E}_k}{t} \right)
\]

\[
= \left( \bar{y}_{km} \right) \bar{E}_m + \left( - \frac{\bar{y}_{km}}{t} \right) \bar{E}_k
\]

• Also

\[
\bar{I}_k = - \frac{1}{t} \bar{I}_{k'} = \left( - \frac{\bar{y}_{km}}{t} \right) \bar{E}_m + \left( \frac{\bar{y}_{km}}{t^2} \right) \bar{E}_k
\]
Transformer Nodal Equations

- We may rewrite these two equations as

\[
\begin{bmatrix}
\bar{I}_k \\
\bar{I}_m
\end{bmatrix}
= \begin{bmatrix}
\frac{\bar{y}_{km}}{t^2} & -\frac{\bar{y}_{km}}{t} \\
-\frac{\bar{y}_{km}}{t} & \bar{y}_{km}
\end{bmatrix}
\begin{bmatrix}
\bar{E}_k \\
\bar{E}_m
\end{bmatrix}
\]

\(\bar{Y}_{bus}\) is still symmetric here (though this will change with phase shifters)

The $\pi$-Equivalent Circuit for a Transformer Branch

\[
\bar{y}_{km} \left( \frac{1}{t^2} - \frac{1}{t} \right)
\]

\[
\bar{y}_{km} \left( 1 - \frac{1}{t} \right)
\]
Variable Tap Voltage Control

- A transformer with a variable tap, i.e., the variable $t$ is not constant, may be used to control the voltage at either the bus on the side of the tap or at the bus on the side away from the tap.
- This constitutes an example of single criterion control since we adjust a single control variable (i.e., the transformer tap $t$) to achieve a specified criterion: the maintenance of a constant voltage at a designated bus.
- Names for this type of control are on-load tap changer (LTC) transformer or tap changing under load (TCUL).
- Usually on low side; there may also be taps on high side that can be adjusted when it is de-energized.
Variable Tap Voltage Control

• An LTC is a discrete control, often with 32 incremental steps of 0.625% each, giving an automatic range of ±10%

• It follows from the π–equivalent model for the transformer that the transfer admittance between the buses of the transformer branch and the contribution to the self admittance at the bus away from the tap explicitly depend on \( t \)

• However, the tap changes in discrete steps; there is also a built in time delay in how fast they respond

• Voltage regulators are devices with a unity nominal ratio, and then a similar tap range
Ameren Champaign (IL) Test Facility
Voltage Regulators

These are connected on the low side of a 69/12.4 kV transformer; each phase can be regulated separately.