# ECEN 615
# Methods of Electric Power Systems Analysis

## Lecture 3: Per Unit, Ybus, Power Flow

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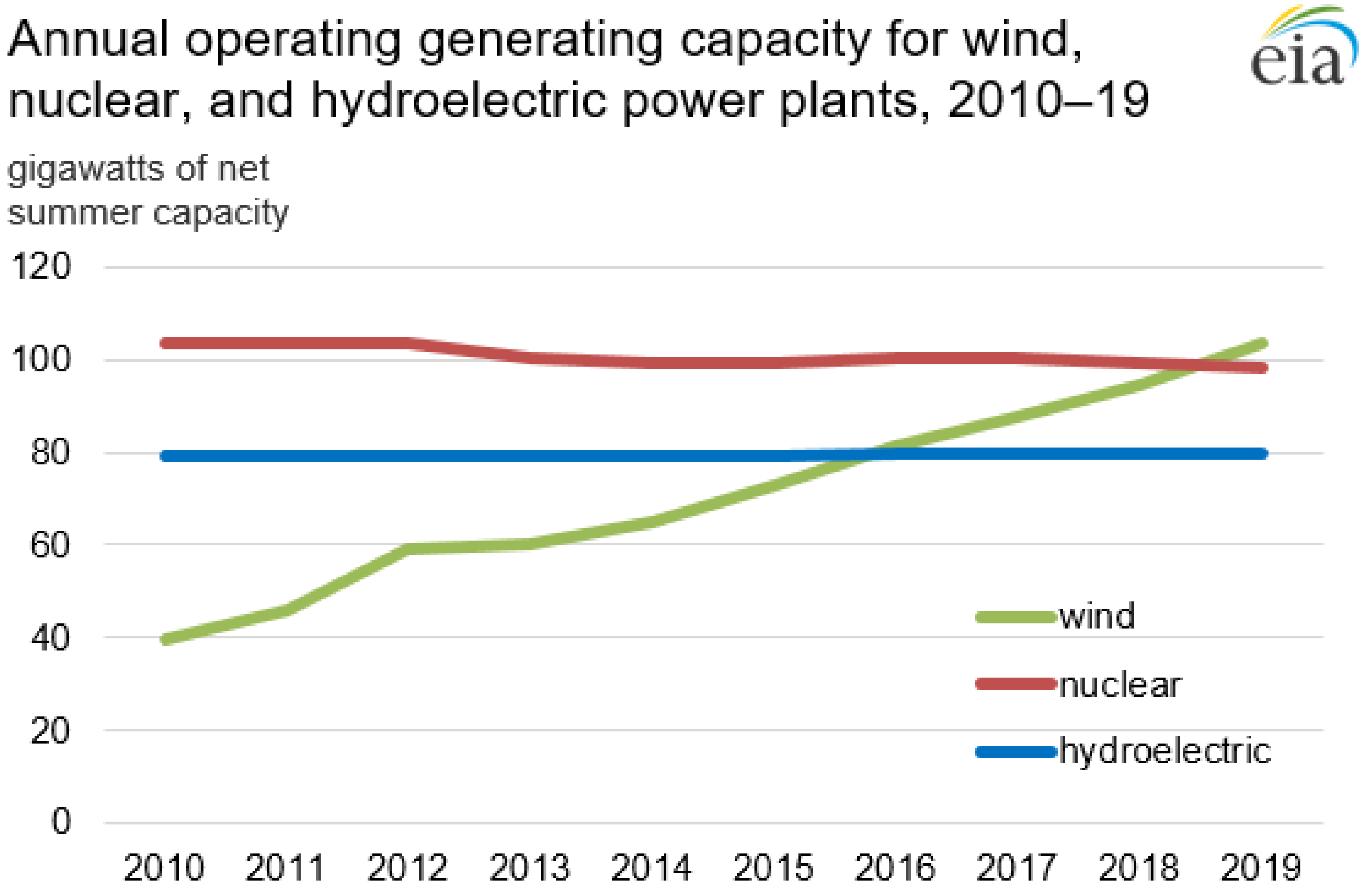
# Announcements

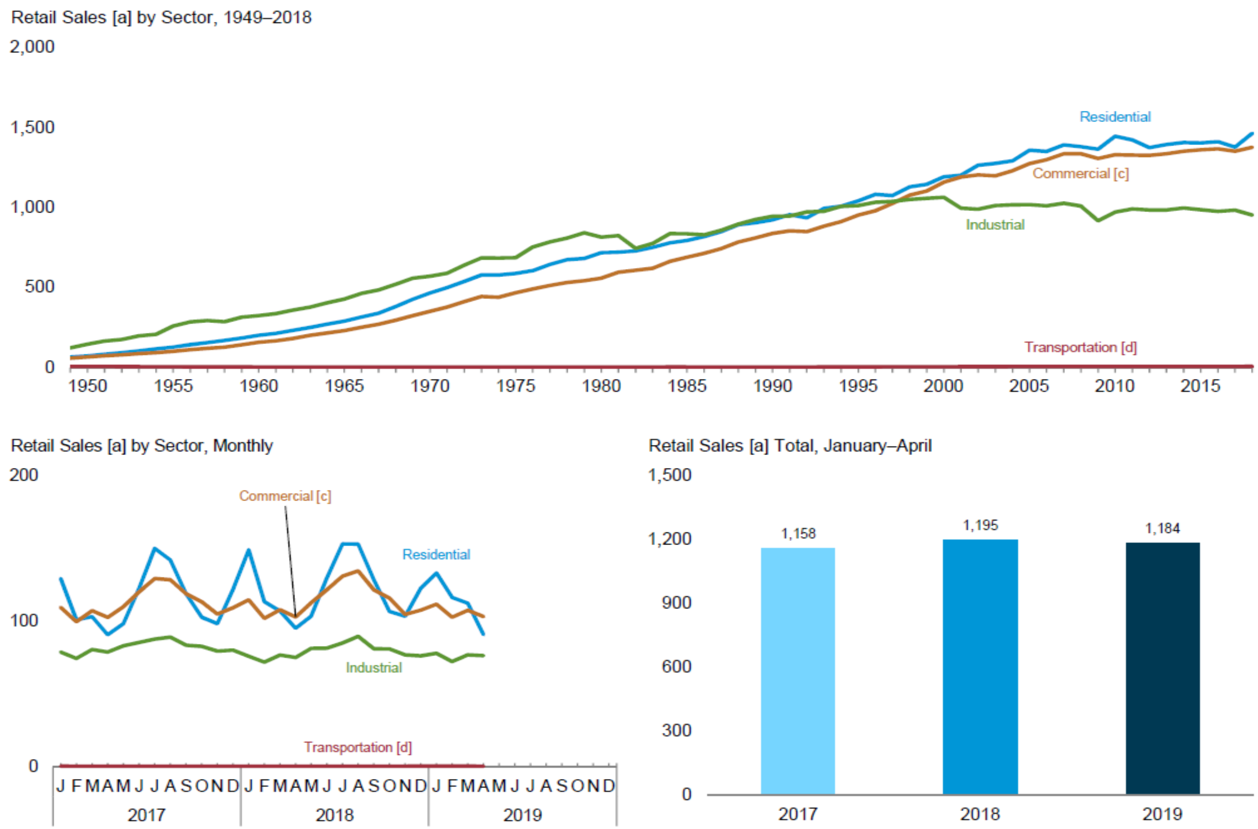
- Start reading Chapters 1 to 3 from the book (mostly background material)
- Homework 1 is assigned today. It is due on Thursday September 3
- Public website is overbye.engr.tamu.edu/ecen-615-fall-2020/

# Wind now surpasses nuclear and hydro




Source: www.eia.gov/electricity/monthly/update/ (April 2020)

# Slowing Electric Load Growth



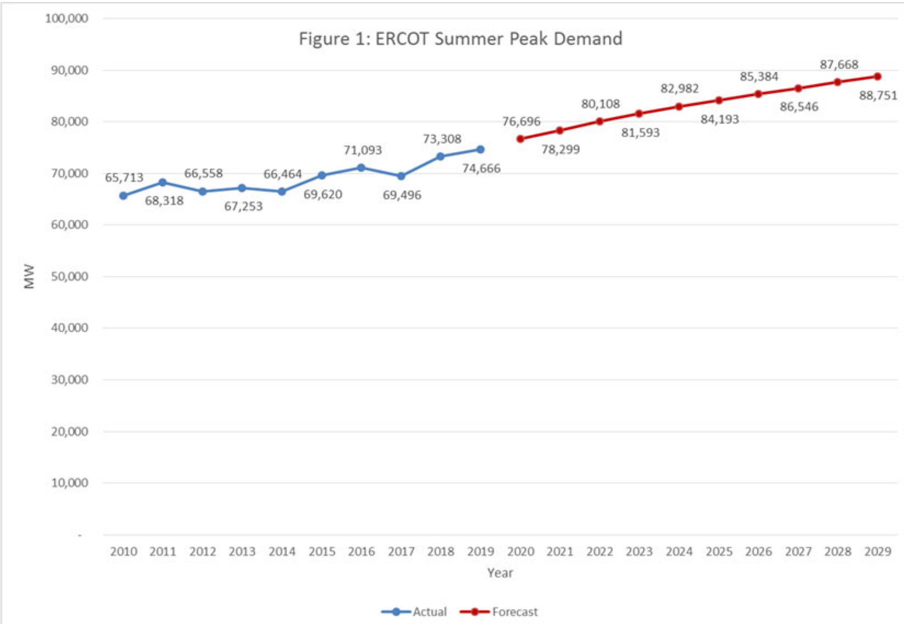

[a] Electricity retail sales to ultimate customers reported by utilities and other energy service providers.
[b] See "Direct Use" in Glossary.
[c] Commercial sector, including public street and highway lighting, interdepartmental sales, and other sales to public authorities.
[d] Transportation sector, including sales to railroads and railways.
Web Page: http://www.eia.gov/totalenergy/data/monthly/#electricity.
Source: Table 7.6.

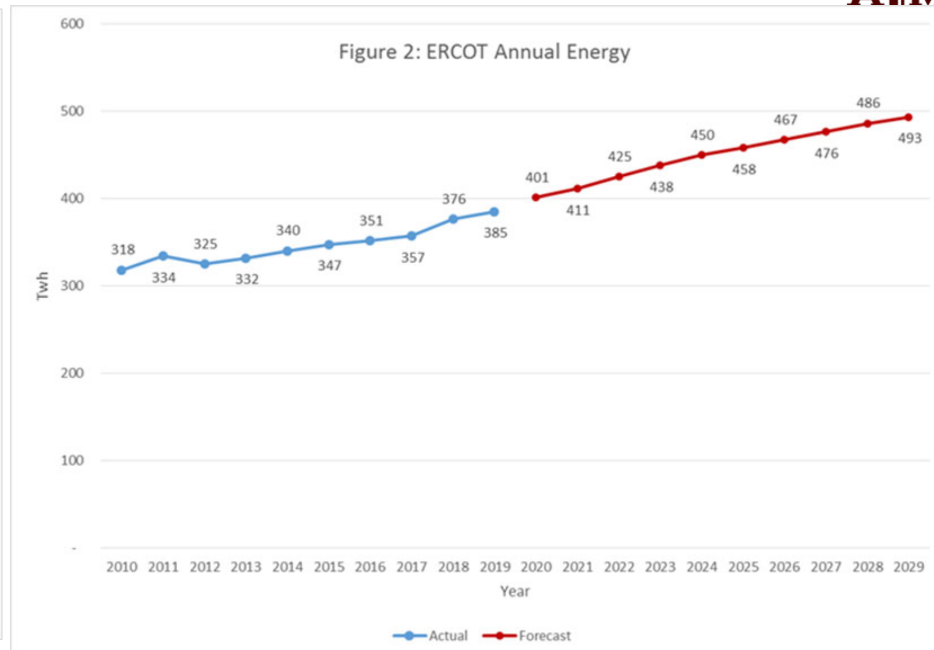
138 U. S. Energy Information Administration / Monthly Energy Review July 2019

Much of the slowing load growth is due to distributed generation, such as solar PV, which sits on the customer side of the meter

Source: EIA Monthly Energy Review, July 2019

# Except in Texas!




The left graph is peak demand, the right energy

ERCOT set a new peak electric load of 74.5 GW on 8/12/19, surpassing the 73.3 GW record from 2018; total energy in 2017 was 357 billion kWh

Source: www.ercot.com/gridinfo/load/forecast

# Interconnected Power System Basic Characteristics

- Three – phase AC systems:
  - generation and transmission equipment is usually three phase
  - industrial loads are three phase
  - residential and commercial loads are single phase and distributed equally among the phases; consequently, a balanced three – phase system results
- Synchronous machines generate electricity
  - Exceptions: some wind is induction generators; solar PV
- Interconnection transmits power over a wider region with subsystems operating at different voltage levels

# Load Models (Omitted from Lecture 2)

- Ultimate goal is to supply loads with electricity at constant frequency and voltage
- Electrical characteristics of individual loads matter, but usually they can only be estimated
  - actual loads are constantly changing, consisting of a large number of individual devices
  - only limited network observability of load characteristics
- Aggregate models are typically used for analysis
- Two common models
  - constant power: $S_i = P_i + jQ_i$
  - constant impedance: $S_i = |V|^2 / Z_i$

The ZIP model combines constant impedance, current and power (P)

# Reading Technical Papers

- As a graduate student you should get in the habit of reading many technical papers, including all the ones mentioned in these notes
- Papers are divided into 1) journal papers and 2) conference papers, with journal papers usually undergoing more review and of high quality
  - There are LOTs of exceptions
- Key journals in our area are from IEEE Power and Energy Society (PES); PSCC is a top conference
- I read papers by looking at 1) title, 2) abstract, 3) summary, 4) results, 5) intro, 6) the rest; many papers never make it beyond step 1 or 2.

# Learn to Write Well

- Writing is a key skill for engineers, especially for students with advanced degrees
- If you are not currently a good technical writer, use your time at TAMU to learn how to write well!
- There are lots of good resources available to help you improve your writing.  Books I've found helpful are
  - Zinsser, "On Writing Well: The Classic Guide to Writing Nonfiction"
  - Strunk and White, "The Elements of Style"
  - Alred, Oliu, Brusaw, "The Handbook of Technical Writing"
- TAMU Writing Center, writingcenter.tamu.edu/

# Three-Phase Per Unit

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system, $S_B^{3\phi}$
2. Pick a voltage base for each different voltage level, $V_B$. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

# Three-Phase Per Unit, cont'd

4. Calculate the current base, $I_B$

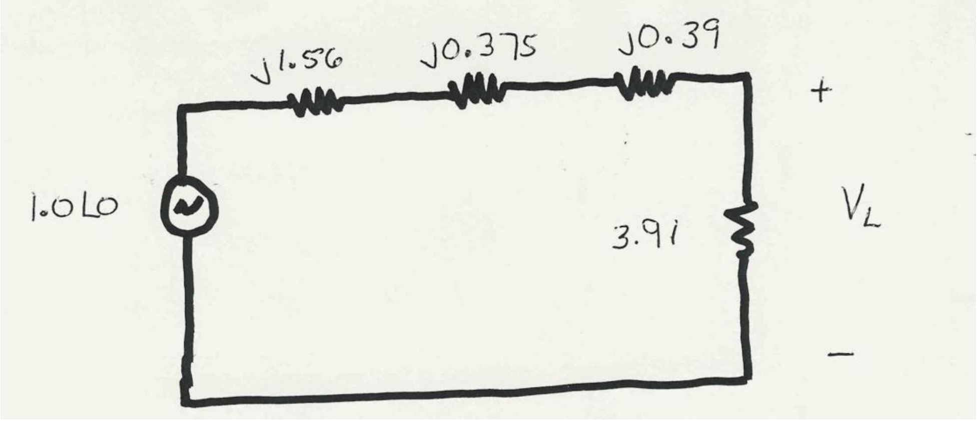
$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B,LL}} = \frac{3\,S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\,V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

Exactly the same current bases as with single-phase!

5. Convert actual values to per unit

# Three-Phase Per Unit Example

Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ power base of **300 MVA**, and line to line voltage bases of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line to line voltage is 13.8 kV.




Convert to per unit as before. Note the system is exactly the same!

# Three-Phase Per Unit Example, cont.

$$I = \frac{1.0\angle 0°}{3.91 + j2.327} = 0.22\angle -30.8° \text{ p.u. (not amps)}$$

$$V_L = 1.0\angle 0° - 0.22\angle -30.8° \times 2.327\angle 90°$$

$$= 0.859\angle -30.8° \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0° \times 0.22\angle 30.8° = 0.22\angle 30.8° \text{ p.u.}$$

Again, analysis is exactly the same!

# Three-Phase Per Unit Example, cont'd

Differences appear when we convert back to actual values

$$V_L^{\text{Actual}} = 0.859\angle -30.8° \times 27.6 \text{ kV} = 23.8\angle -30.8° \text{ kV}$$

$$S_L^{\text{Actual}} = 0.189\angle 0° \times 300 \text{ MVA} = 56.7\angle 0° \text{ MVA}$$

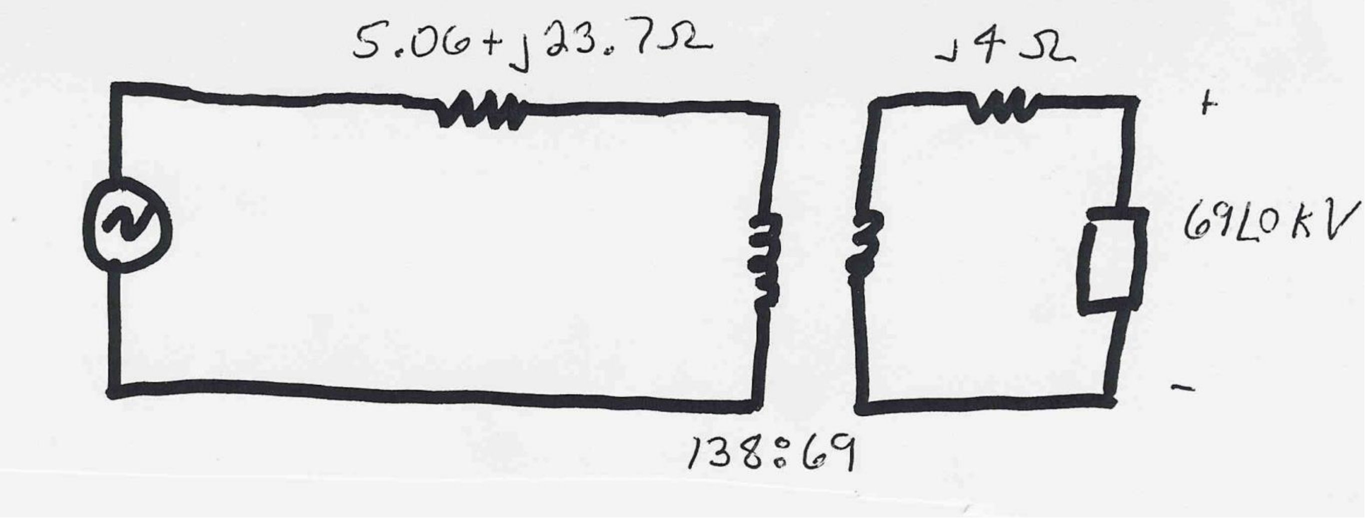
$$S_G^{\text{Actual}} = 0.22\angle 30.8° \times 300 \text{ MVA} = 66.0\angle 30.8° \text{ MVA}$$

$$I_B^{\text{Middle}} = \frac{300 \text{ MVA}}{\sqrt{3}\,138 \text{ kV}} = 1250 \text{ Amps} \quad \text{(same current!)}$$

$$I_{\text{Middle}}^{\text{Actual}} = 0.22\angle -30.8° \times 1250 \text{ Amps} = 275\angle -30.8° \text{ A}$$

# Three-Phase Per Unit Example 2

- Assume a 3ϕ load of 100+j50 MVA with $V_{LL}$ of 69 kV is connected to a source through the below network:




What is the supply current and complex power?

Answer: I=467 amps, S = 103.3 + j76.0 MVA

# Power Flow Analysis

- We now have the necessary models to start to develop the power system analysis tools
- The most common power system analysis tool is the power flow (also known sometimes as the load flow)
  - power flow determines how the power flows in a network
  - also used to determine all bus voltages and all currents
  - because of constant power models, power flow is a nonlinear analysis technique
  - power flow is a steady-state analysis tool

# Linear versus Nonlinear Systems

A function **H** is linear if

$$\mathbf{H}(\alpha_1\boldsymbol{\mu}_1 + \alpha_2\boldsymbol{\mu}_2) = \alpha_1\mathbf{H}(\boldsymbol{\mu}_1) + \alpha_2\mathbf{H}(\boldsymbol{\mu}_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

**Linear Example:** $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2) = c\mathbf{x}_1 + c\,\mathbf{x}_2$$

**Nonlinear Example:** $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}^2$

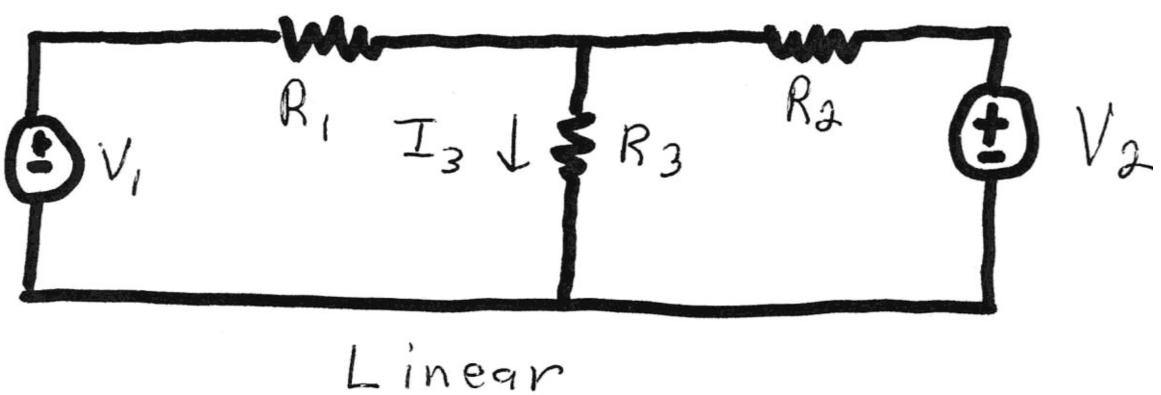
$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

# Linear Power System Elements

Resistors, inductors, capacitors, independent voltage sources and current sources are linear circuit elements

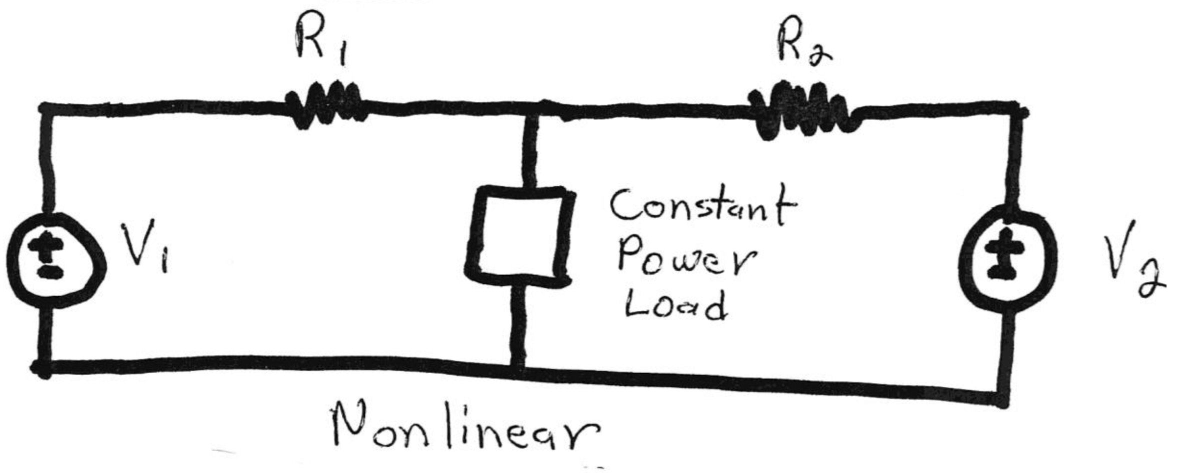
$$V = R\,I \qquad V = j\omega L\,I \qquad V = \frac{1}{j\omega C} I$$

Such systems may be analyzed by superposition



# Nonlinear System Example

- Constant power loads and generator injections are nonlinear and hence systems with these elements can not be analyzed by superposition



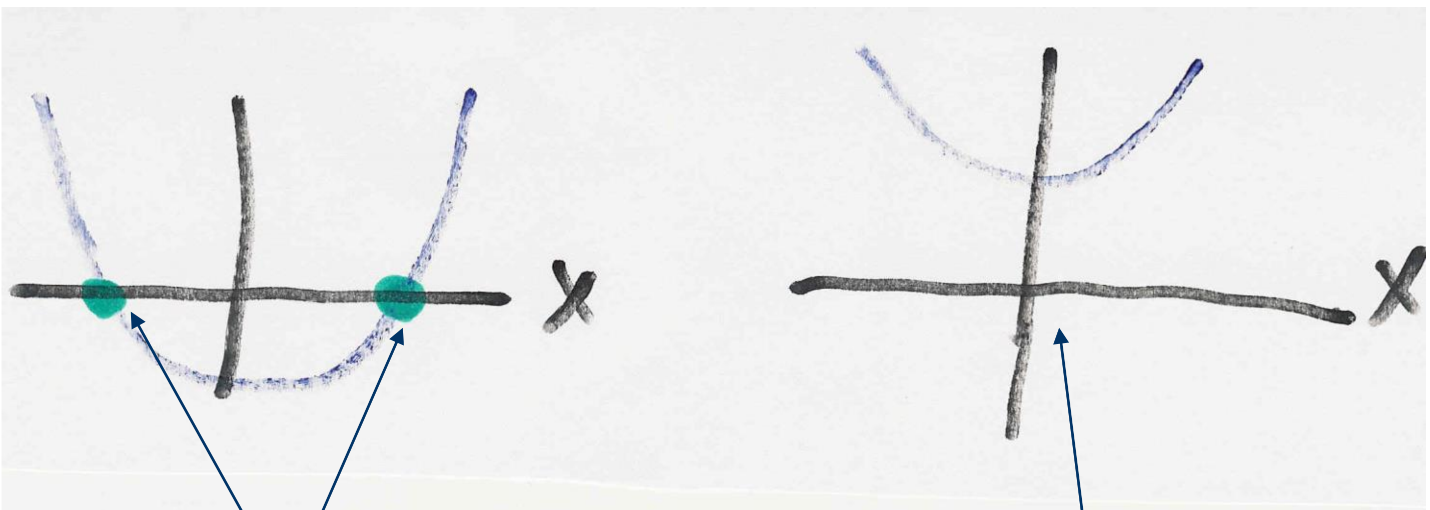

Nonlinear problems can be very difficult to solve, and usually require an iterative approach

# Nonlinear Systems May Have Multiple Solutions or No Solution

Example 1: $x^2 - 2 = 0$ has solutions $x = \pm 1.414\ldots$

Example 2: $x^2 + 2 = 0$ has no real solution

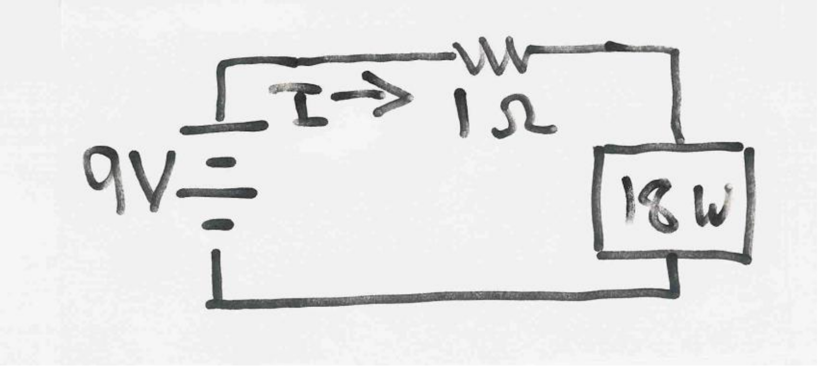
$f(x) = x^2 - 2$ $\quad\quad$ $f(x) = x^2 + 2$




two solutions where $f(x) = 0$ $\quad\quad$ no solution $f(x) = 0$

# Multiple Solution Example

- The dc system shown below has two solutions:




where the 18 watt load is a resistive load

The equation we're solving is

$$I^2 R_{Load} = \left( \frac{9 \text{ volts}}{1\Omega + R_{Load}} \right)^2 R_{Load} = 18 \text{ watts}$$

One solution is $R_{Load} = 2\Omega$

Other solution is $R_{Load} = 0.5\Omega$

What is the maximum $P_{Load}$?

# Bus Admittance Matrix or $Y_{bus}$

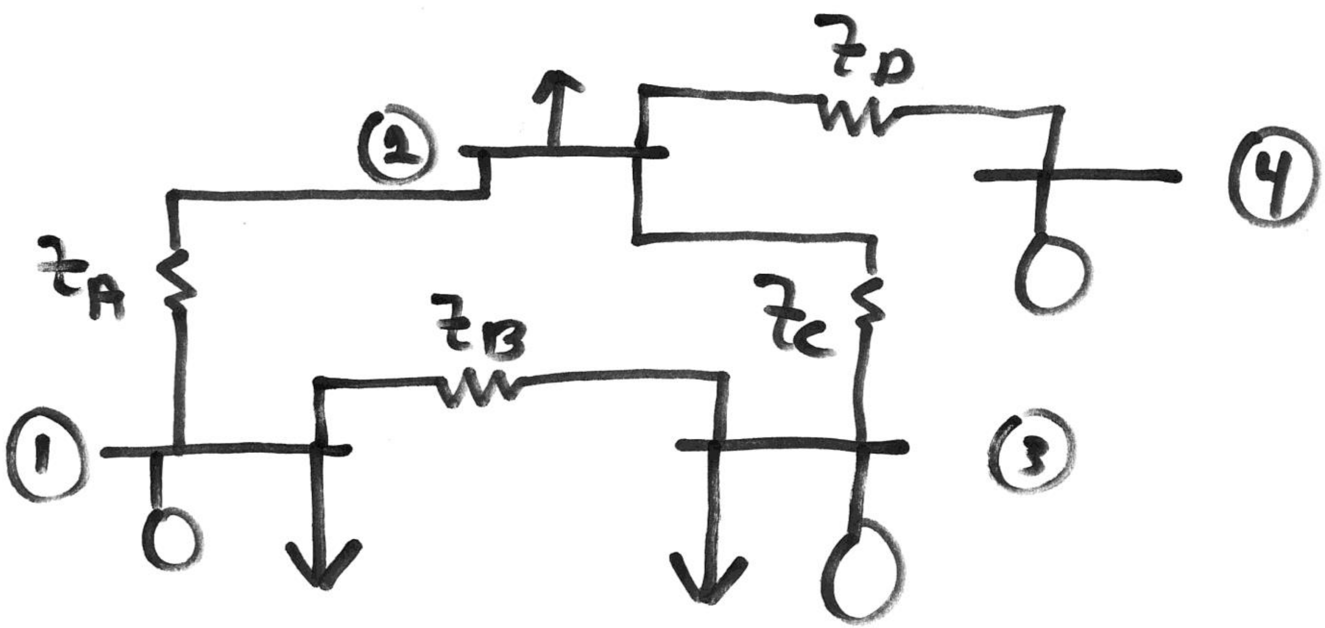
- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, **I**, and all the bus voltages, **V**,

$$\mathbf{I} = \mathbf{Y}_{bus} \mathbf{V}$$

- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

# $Y_{bus}$ Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus i is $I_i = I_{Gi} - I_{Di}$ where $I_{Gi}$ is the current injection into the bus from the generator and $I_{Di}$ is the current flowing into the load



# $Y_{bus}$ Example, cont'd

By KCL at bus 1 we have

$$I_1 = I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{13} = \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$I_1 = (V_1 - V_2)Y_A + (V_1 - V_3)Y_B \qquad (\text{with } \mathrm{Y_j} = \frac{1}{Z_j})$$

$$= (Y_A + Y_B)V_1 - Y_A V_2 - Y_B V_3$$

Similarly

$$I_2 = I_{21} + I_{23} + I_{24}$$

$$= -Y_A V_1 + (Y_A + Y_C + Y_D)V_2 - Y_C V_3 - Y_D V_4$$

# $Y_{bus}$ Example, cont'd

We can get similar relationships for buses 3 and 4

The results can then be expressed in matrix form

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$$

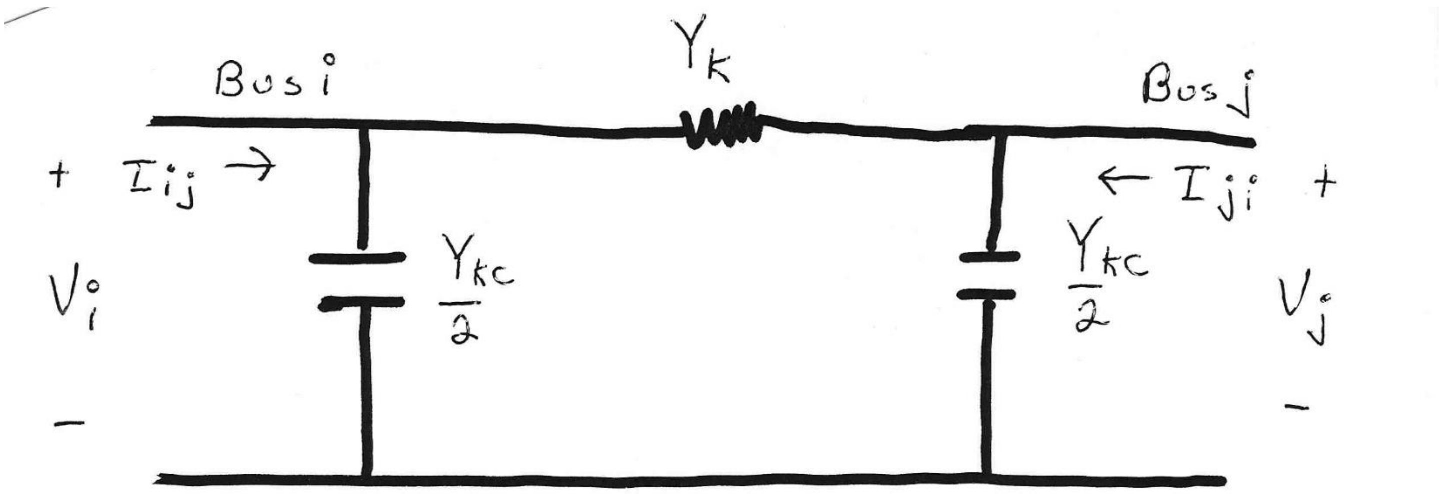
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For a system with n buses the $Y_{bus}$ is an n by n symmetric matrix (i.e., one where $A_{ij} = A_{ji}$); however this will not be true in general when we consider phase shifting transformers

# $Y_{bus}$ General Form

- The diagonal terms, $Y_{ii}$, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, $Y_{ij}$, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems $Y_{bus}$ is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the $\pi$ line model, only affect the diagonal terms.

# Modeling Shunts in the $Y_{bus}$

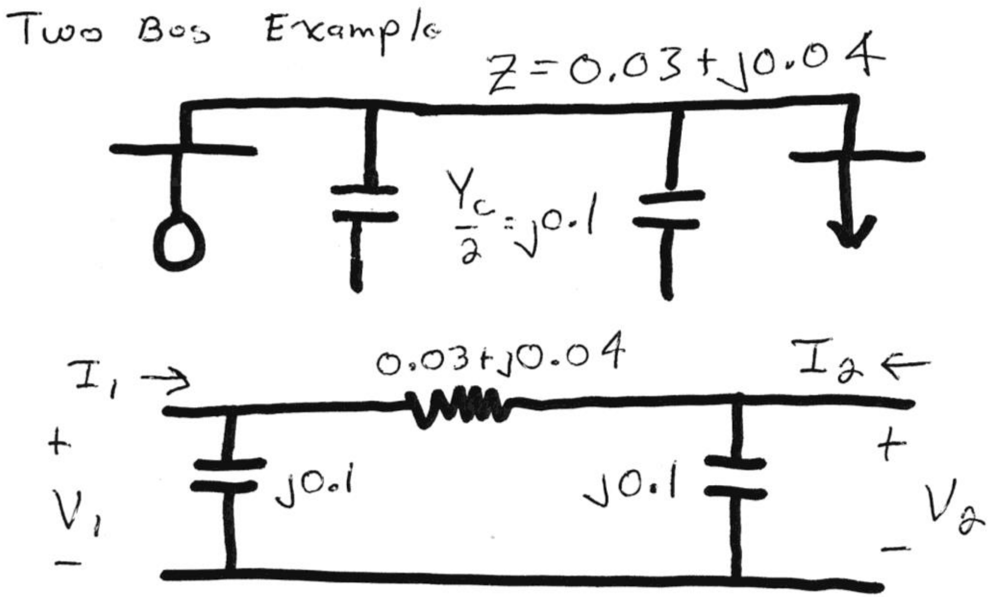



Since $I_{ij} = (V_i - V_j)Y_k + V_i \frac{Y_{kc}}{2}$

$$Y_{ii} = Y_{ii}^{\text{from other lines}} + Y_k + \frac{Y_{kc}}{2}$$

Note $Y_k = \frac{1}{Z_k} = \frac{1}{R_k + jX_k} \frac{R_k - jX_k}{R_k - jX_k} = \frac{R_k - jX_k}{R_k^2 + X_k^2}$

# Two Bus System Example




$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} \qquad \frac{1}{0.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

# Using the $Y_{bus}$

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the bus impedance matrix

However, this requires that $\mathbf{Y}_{\text{bus}}$ not be singular; note it will be singular if there are no shunt connections!

# Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$S_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$