1. Book problem 9.1 except change M_{12} to be 55 MW and M_{32} to be 35.5 MW. Assume a linear (dc power flow approximation) system model. That is, **f(x)** = **Hx**.

$$M_{12} = \frac{\theta_1 - \theta_2}{0.2};\ M_{13} = \frac{\theta_1 - \theta_3}{0.4};\ M_{32} = \frac{\theta_3 - \theta_2}{0.25};\ H = \begin{bmatrix} 5.0 & -5.0 \\ 2.5 & 0 \\ 0 & -4.0 \end{bmatrix};$$

$$z^{meas} = \begin{bmatrix} 0.55 \\ 0.04 \\ 0.355 \end{bmatrix} p.u.$$

$$\theta_3 = 0$$

$$R = \begin{bmatrix} 4 \times 10^4 & 0 & 0 \\ 0 & 1 \times 10^{-4} & 0 \\ 0 & 0 & 4 \times 10^{-6} \end{bmatrix}$$

$$x^{est} = [H'R^{-1}H]^{-1}H'R^{-1}z^{meas}$$

$$x^{est} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.0186 \\ -0.0888 \end{bmatrix} radians$$

Residual, $J(x) = J(\theta_1, \theta_2) = \frac{[z_{12} - M_{12}(\theta_1,\theta_2)]^2}{\sigma_{12}^2} + \frac{[z_{13} - M_{13}(\theta_1,\theta_3)]^2}{\sigma_{13}^2} + \frac{[z_{32} - M_{32}(\theta_3,\theta_2)]^2}{\sigma_{32}^2}$

= 0.4225 + 0.4225 + 0.01 = 0.855

Degree of freedom (K) = Number of measurements – number of states = 3 – 2 =1

Using a Chi distribution table, for a significant level (α = 0.01) and K = 1, the threshold residual, t_j = 6.635.

Since $J(x) << t_j$, it is safe to assume the likely absence of bad data in the measurements.

2. Book problem 9.3. Again assume a linear system model.

a) Th network is unobservable since there's no known measurements that pertain to bus 4.

$$M_{13} = \frac{\theta_1 - \theta_3}{0.5};\ M_{31} = \frac{\theta_3 - \theta_1}{0.5};\ M_{12} = \frac{\theta_1 - \theta_2}{0.25};\ H = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 4 & -4 & 0 \end{bmatrix}$$

$$z^{meas} = \begin{bmatrix} -0.705 \\ 0.721 \\ 0.212 \end{bmatrix} p.u.;\ \theta_4 = 0$$

$$R = \begin{bmatrix} 1 \times 10^4 & 0 & 0 \\ 0 & 1 \times 10^{-4} & 0 \\ 0 & 0 & 4 \times 10^{-4} \end{bmatrix}$$

$$H^1R^{-1}H = \begin{bmatrix} 12000 & -40000 & -80000 \\ -4000 & 40000 & 0 \\ -8000 & 0 & 80000 \end{bmatrix}$$

$H^1R^{-1}H$ is singular (hence, noninvertible) since the system is unobservable.

b) $H = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 4 & -4 & 0 \\ -2 & 0 & 12 \end{bmatrix};\ R = \begin{bmatrix} 1 \times 10^{-4} & 0 & 0 & 0 \\ 0 & 1 \times 10^{-4} & 0 & 0 \\ 0 & 0 & 4 \times 10^{-4} & 0 \\ 0 & 0 & 0 & 2.25 \times 10^{-4} \end{bmatrix};$

$$z^{meas} = \begin{bmatrix} -0.705 \\ 0.721 \\ 0.212 \\ 0.920 \end{bmatrix} p.u.$$

$$H^1R^{-1}H = \begin{bmatrix} 1.3778 & -0.4000 & -1.8667 \\ -0.4000 & 0.4000 & 0 \\ -1.8667 & 0 & 7.2000 \end{bmatrix} * 10^5$$

$H^1R^{-1}H$ is now full-rank (hence, invertible) and thus observable.

$$x^{est} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -0.3358 \\ -0.3888 \\ 0.0207 \end{bmatrix} radians$$

3. For the two bus example from Lecture 18, assume that $B_{12} = B_{21} = 12$ and that

$$\mathbf{Z}^{meas} = \begin{bmatrix} P_{12} \\ Q_{12} \\ P_{21} \\ Q_{21} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.52 \\ 1.2 \\ -1.48 \\ -1 \\ 1.01 \\ 0.93 \end{bmatrix} \quad x^0 = \begin{bmatrix} V_1 \\ \theta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \sigma_i = 0.01$$

Do the first iteration.

$$x^1 = \begin{bmatrix} 1.0158 \\ -0.125 \\ 0.924 \end{bmatrix}$$

4. Using the Givens Rotation algorithm, manually perform a QR factorization of the matrix given below. Show your work at each step.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 5 & 6 \end{bmatrix}$$

First, we zero out A[3,1], a = 4, b = 5, gives s = -0.7808, c = 0.6247

$$G_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.6247 & -0.7808 \\ 0 & 0.7808 & 0.6247 \end{bmatrix}, G_1{}^T A = \begin{bmatrix} 2 & 1 \\ 6.4028 & 6.5589 \\ 0 & 1.4058 \end{bmatrix}$$

Next, zero out A[2,1], a = 2, b = 6.4028, gives s = -0.9545 , c = 0.2982

$$G_2 = \begin{bmatrix} 0.2982 & -0.9545 & 0 \\ 0.9545 & 0.2982 & 0 \\ 0 & 0 & 1 \end{bmatrix}, G_2^T G_1^T A = \begin{bmatrix} 6.7079 & 6.5587 \\ 0 & 1.0014 \\ 0 & 1.4058 \end{bmatrix}$$

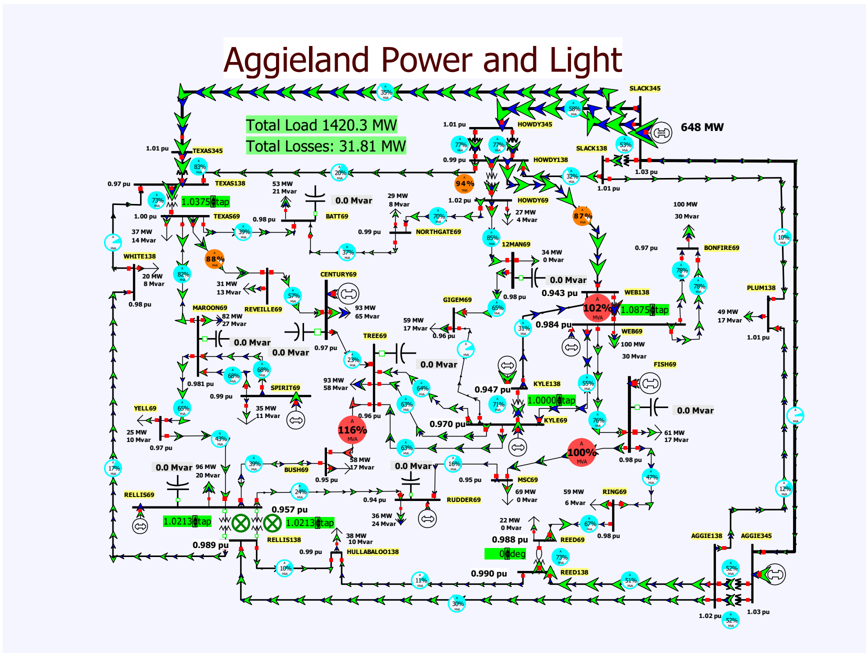
Then, zero out A[3,2], a = 1.0014, b= 1.4058, gives s = -0.8145, c = 0.5801

$$G_3 = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 0.5801 & -0.8145 \\ 0 & 0.8145 & 0.5801 \end{bmatrix}, G_3^T G_2^T G_1^T A = \begin{bmatrix} 6.7079 & 6.5587 \\ 0 & 1.7259 \\ 0 & 0 \end{bmatrix}$$

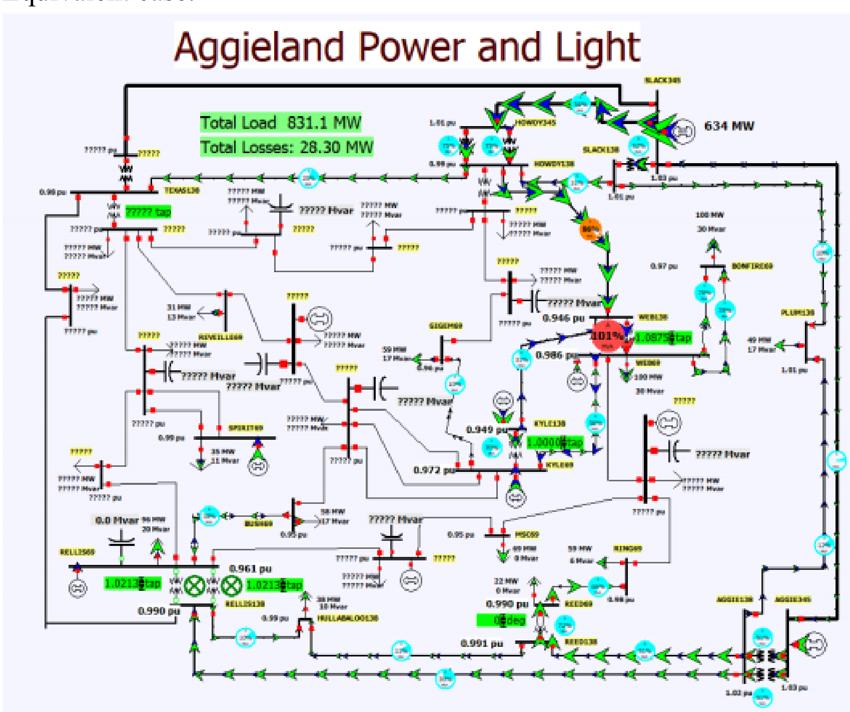
Thus, $G_1 G_2 G_3 G_3^T G_2^T G_1^T A = QU = \begin{bmatrix} 0.2982 & -0.5537 & 0.7774 \\ 0.5963 & -0.5279 & -0.6047 \\ 0.7453 & 0.6439 & 0.1727 \end{bmatrix} \begin{bmatrix} 6.7079 & 6.5587 \\ 0 & 1.7259 \\ 0 & 0 \end{bmatrix}$

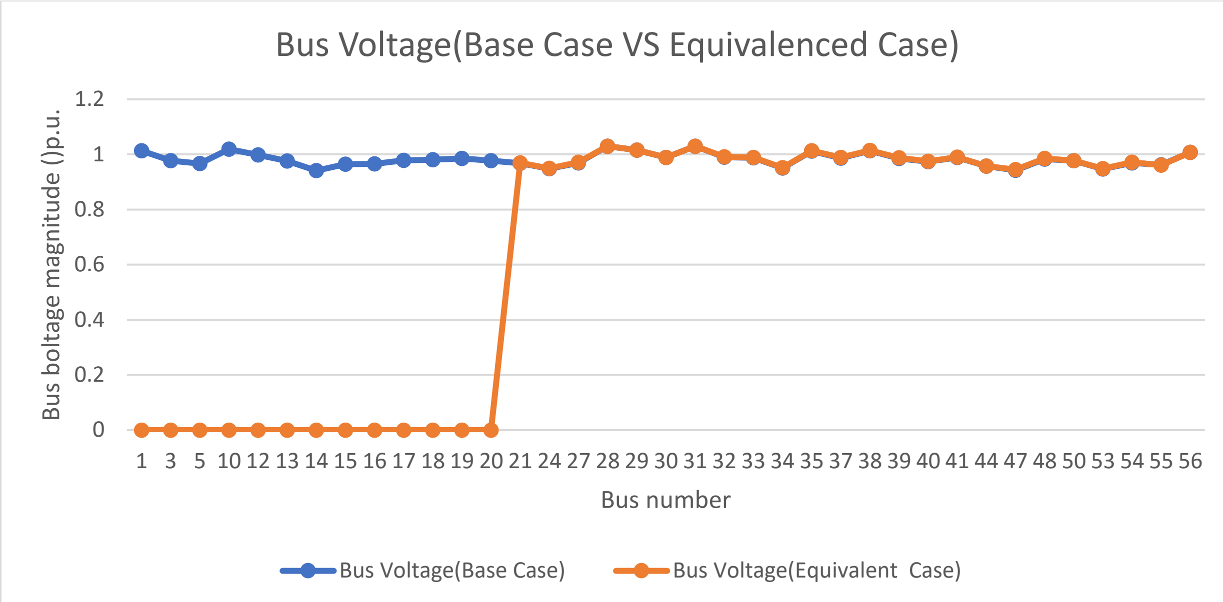
5. In PowerWorld Simulator using the Aggieland37_HW5 case, first calculate the line flows and bus voltage magnitudes for the contingent opening of both of the transformers between buses 41 and 44. You may wish to store these results in a spreadsheet. Then, reopen the case (i.e., without the contingency) and in PowerWorld create an equivalent eliminating all the buses with bus numbers less than 21. Then, repeat the previous contingency, and compare the results with the full system (obviously only comparing for the retained buses and lines).

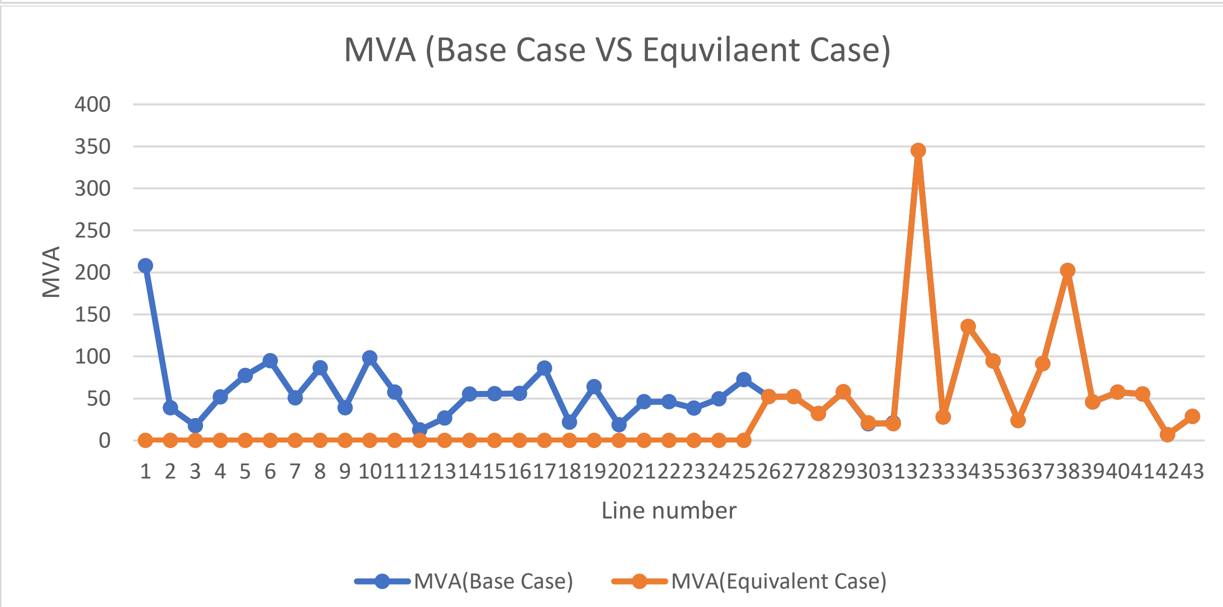
   Base case:



   Equivalent case:







Barring small differences, the values for the power system states – both bus voltage magnitude and line flows – are similar in the base case and the equivalent case. It shows the close similarity of the equivalenced, 24-bus case and the actual, 37-bus case. Also, the line overload between WEB138-WEB69 is retained in the smaller, 24-bus case. Selected states of large-scale systems can be easily and quickly analyzed if smaller, equivalent systems (which retains original grid dynamics and containing the region of interest) are used.

6. In PowerWorld Simulator open the 2000 bus case from Homework 4. First, with the existing three line open contingencies, open the line between buses 3048 and 5045, circuit 1 (this is one of the lines immediately to the west of the line loaded initially at 149%), and note the change in the system flows. Now reopen the Homework 4 case and develop an equivalent that you think well approximates the response of the original system to this contingency. Justify the buses you selected to include in your equivalent, and justify that it gives approximately the same response for this contingency.

   Equivalent system will have similar system flows.