ECEN 615
Methods of Electric Power Systems Analysis
Lecture 18: State Estimation

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Announcements

• Read Chapter 9 from the book
• Homework 4 is due on Thursday October 31.
Nonlinear Formulation

- A regular ac power system is nonlinear, so we need to use an iterative solution approach. This is similar to the Newton power flow. Here assume m measurements and n state variables (usually bus voltage magnitudes and angles) Then the Jacobian is the $H$ matrix

$$H(x) = \frac{\partial f(x)}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{x_1} & \ldots & \frac{\partial f_1}{x_n} \\ \frac{\partial f_m}{x_1} & \ldots & \frac{\partial f_m}{x_n} \end{bmatrix}$$
Assume we measure the real and reactive power flowing into one end of a transmission line; then the $z_i$-$f_i(x)$ functions for these two are

$$P_{ij}^{\text{meas}} = \left[ -V_i^2 G_{ij} + V_i V_j \left( G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right) \right]$$

$$Q_{ij}^{\text{meas}} = \left[ V_i^2 \left( B_{ij} + \frac{B_{\text{cap}}}{2} \right) + V_i V_j \left( G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right) \right]$$

- Two measurements for four unknowns

Other measurements, such as the flow at the other end, and voltage magnitudes, add redundancy
SE Iterative Solution Algorithm

• We then make an initial guess of $x$, $x^{(0)}$ and iterate, calculating $\Delta x$ each iteration.

$$\Delta x = \left[ H^T R^{-1} H \right]^{-1} H^T R^{-1} \begin{bmatrix} z_1 - f_1(x) \\ \vdots \\ z_m - f_m(x) \end{bmatrix}$$

$$x^{(k+1)} = x^{(k)} + \Delta x$$

Keep in mind that $H$ is no longer constant, but varies as $x$ changes. Often ill-conditioned.

This is exactly the least squares form developed earlier with $H^T R^{-1} H$ an $n$ by $n$ matrix. This could be solved with Gaussian elimination, but this isn't preferred because the problem is often ill-conditioned.
Nonlinear SE Solution Algorithm, Book Figure 9.11

9.4 STATE ESTIMATION OF AN AC NETWORK

START

READ MEASUREMENTS

WHERE \( x = \begin{bmatrix} \epsilon \\ \delta \end{bmatrix} \)

PICK STARTING VALUE FOR \( x = x^0 \)

SOLVE FOR \( (z_i - f_i(x)) \) FOR \( i = 1 \cdots N_m \)

CALCULATE H MATRIX AS FUNCTION OF \( x \)

CALCULATE \( H^T R^{-1} H \) MATRIX

CALCULATE \( [H^T R^{-1} H]^{-1} \)

SOLVE FOR \( \Delta x \)

\[
\Delta x = [H^T R^{-1} H]^{-1} H^T R^{-1} \begin{bmatrix} z_1 - f_1(x) \\ z_2 - f_2(x) \end{bmatrix}
\]

CALC MAX \( (|\Delta x_i|) \) \( i = 1 \cdots N_x \)

\[ \text{MAX } (|\Delta x_i|) < \epsilon \]

YES

DONE

NO

UPDATE \( x : x = x + \Delta x \)

FIGURE 9.11 State estimation solution algorithm.
Example: Two Bus Case

• Assume a two bus case with a generator supplying a load through a single line with $x=0.1$ pu. Assume measurements of the p/q flow on both ends of the line (into line positive), and the voltage magnitude at both the generator and the load end. So $B_{12} = B_{21}=10.0$

\[
P_{ij}^{meas} = \left[ V_i V_j \left( B_{ij} \sin(\theta_i - \theta_j) \right) \right]
\]

\[
Q_{ij}^{meas} = \left[ V_i^2 B_{ij} + V_i V_j \left( -B_{ij} \cos(\theta_i - \theta_j) \right) \right]
\]

\[
V_{i}^{meas} - V_i = 0
\]

We need to assume a reference angle unless we directly measuring phase
Example: Two Bus Case

- Let

\[
Z_{\text{meas}} = \begin{bmatrix}
P_{12} \\
Q_{12} \\
P_{21} \\
Q_{21} \\
P_1 \\
Q_1 \\
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
2.02 \\
1.5 \\
-1.98 \\
-1 \\
1.01 \\
0.87
\end{bmatrix}
\]

\[
x^0 = \begin{bmatrix}
V_1 \\
\theta_2 \\
V_2
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}, \sigma_i = 0.01
\]

We assume an angle reference of \( \theta_1=0 \)

\[
H(x) = \begin{bmatrix}
V_2 10 \sin(-\theta_2) & -V_1 V_2 10 \cos(-\theta_2) & V_1 10 \sin(-\theta_2) \\
20V_1 - V_2 10 \cos(-\theta_2) & -V_1 V_2 10 \sin(-\theta_2) & -V_1 10 \cos(-\theta_2) \\
V_2 10 \sin(\theta_2) & V_1 V_2 10 \cos(\theta_2) & V_1 10 \sin(\theta_2) \\
-V_2 10 \cos(\theta_2) & V_1 V_2 10 \sin(\theta_2) & 20V_2 - V_1 10 \cos(\theta_2)
\end{bmatrix}
\]
Example: Two Bus Case

• With a flat start guess we get

\[ H(x^0) = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad z - f(x^0) = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix} \]

\[ R = \begin{bmatrix} 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix} \]
Example: Two Bus Case

\[
H^T R^{-1} H = 1e^6 \times \begin{bmatrix}
2.01 & 0 & -2 \\
0 & 2 & 0 \\
-2 & 0 & 2.01 \\ 
\end{bmatrix}
\]

\[
x^1 = x^0 + \left[ H^T R^{-1} H \right]^{-1} H^T R^{-1} \begin{bmatrix}
2.02 \\
1.5 \\
-1.98 \\
-1 \\
0.01 \\
0.13 \\ 
\end{bmatrix} = \begin{bmatrix}
1.003 \\
-0.2 \\
0.8775 \\ 
\end{bmatrix}
\]
Assumed SE Measurement Accuracy

- The assumed measurement standard deviations can have a significant impact on the resultant solution, or even whether the SE converges.
- The assumption is a Gaussian (normal) distribution of the error with no bias.
In order to estimate all \( n \) states we need at least \( n \) measurements. However, where the measurements are located is also important, a topic known as observability:

- In order for a power system to be fully observable usually we need to have a measurement available no more than one bus away.
- At buses we need to have at least measurements on all the injections into the bus except one (including loads and gens).
- Loads are usually flows on feeders, or the flow into a transmission to distribution transformer.
- Generators are usually just injections from the GSU.
Pseudo Measurements

- Pseudo measurements are used at buses in which there is no load or generation; that is, the net injection into the bus is known with high accuracy to be zero.
  - In order to enforce the net power balance at a bus, we need to include an explicit net injection measurement.

- To increase observability, sometimes estimated values are used for loads, shunts, and generator outputs.
  - These “measurements” are represented as having a higher much standard deviation.
SE Observability Example
SE Bad Data Detection

- The quality of the measurements available to an SE can vary widely, and sometimes the SE model itself is wrong. Causes include
  - Modeling Errors: perhaps the assumed system topology is incorrect, or the assumed parameters for a transmission line or transformer could be wrong
  - Data Errors: measurements may be incorrect because of incorrect data specifications, like the CT ratios or even flipped positive and negative directions
  - Transducer Errors: the transducers may be failing or may have bias errors
  - Sampling Errors: SCADA does not read all values simultaneously and power systems are dynamic
SE Bad Data Detection

- The challenge for SE is to determine when there is likely a bad measurement (or multiple ones), and then to determine the particular bad measurements.

- $J(x)$ is random number, with a probability density function (PDF) known as a chi-squared distribution, $\chi^2(K)$, where $K$ is the degrees of freedom, $K=m-n$.

- It can be shown the expected mean for $J(x)$ is $K$, with a standard deviation of $\sqrt{2K}$.
  - Values of $J(x)$ outside of several standard deviations indicate possible bad measurements, with the measurement residuals used to track down the likely bad measurements.

- SE can be re-run without the bad measurements.
Example SE Application: PJM and MISO

- PJM provides information about their EMS model in
  - [www.pjm.com/-/media/documents/manuals/m03a.ashx](http://www.pjm.com/-/media/documents/manuals/m03a.ashx)

Data here is from the Sept 2018 (Rev 16) document
Example SE Application: PJM and MISO

- PJM measurements are required for 69 kV and up
- PJM SE is triggered to execute every minute
- PJM SE solves well over 98% of the time
- Below reference provides info on MISO SE from March 2015
  - 54,433 buses
  - 54,415 network branches
  - 6332 generating units
  - 228,673 circuit breakers
  - 289,491 mapped points

Energy Management Systems (EMSs)

- EMSs are now used to control most large scale electric grids
- EMSs developed in the 1970’s and 1980’s out of SCADA systems
  - An EMS usually includes a SCADA system; sometimes called a SCADA/EMS
- Having a SE is almost the definition of an EMS. The SE then feeds data to the more advanced functions
- EMSs have evolved as the industry as evolved as the industry has evolved, with functionality customized for the application (e.g., a reliability coordinator or a vertically integrated utility)
EEI Member Companies

EEI U.S. Member Company Service Territories

Electric Coops

America’s Electric Cooperative Network

[Map of Electric Cooperatives across the United States]
ERCOT Control Center with EMS
ERCOT EMS

EMS Applications

- Load and Wind Forecasting
- Load Frequency Control
- Resource Limit Calculator
- State Estimator
- Real-Time Contingency Analysis (RTCA)
- Transmission Constraint Manager (TCM)
- Dynamic Ratings
- Forced Outage Detection

EMS->MMS Interface

MMS Applications

- Security Constrained Economic Dispatch (SCED)
- Ancillary Service (AS) Manager
- Reliability Unit Commitment (WRUC, DRUC, HRUC)
- Supplemental Ancillary Services Market (SASM)
- Look Ahead SCED (LASCED)
- Day Ahead Market (DAM)