ECEN 615 Problem Set #1 Fall 2019
Due 9/12/19

1. Use the Newton-Raphson method to find one solution to the polynomial equation $f(x) = x^3 - 9x^2 - 14x - 30 = 0$. Start with an intial guess of 0 and continue until the mismatch is below a tolerance of $\varepsilon = 0.001$.

NR update method:

$$x^{(new)} = x^{(old)} - \frac{f(x^{(old)})}{f'(x^{(old)})}$$

$$f(x) = x^3 - 9x^2 - 14x - 30$$
$$f'(x) = 3x^2 - 18x - 14$$
$$x^{ini} = 0, \quad \varepsilon = 0.001 \text{ (Mismatch)}$$

First iteration:

$$x^{(0)} = x^{ini} = 0$$
$$f(x^{(0)}) = -30$$
$$f'(x^{(0)}) = -14$$
$$x^{(1)} = x^{(0)} - \frac{f(x^{(0)})}{f'(x^{(0)})} = 0 - \left(\frac{-30}{-14}\right) = -2.143$$

Check: $f(x^{(1)}) = (-2.143)^3 - 9(-2.143)^2 - 14(-2.143) - 30 = -51.17$

$|f(x^{(1)})| = 51.17 > \varepsilon$, hence do Second iteration

Second iteration

$$x^{(2)} = x^{(1)} - \frac{f(x^{(1)})}{f'(x^{(1)})}$$

$$f(x^{(1)}) = -51.17$$
$$f'(x^{(1)}) = +38.35$$
$$x^{(2)} = -2.143 - \left(\frac{-51.17}{+38.35}\right) = \cancel{-2.456}\ -0.809$$

Check: $f(x^{(2)}) = (-0.809)^3 - 9(-0.809)^2 - 14(-0.809) - 30 = -25.09$

$|f(x^{(2)})| = 25.09 > \varepsilon$, hence do third iteration

Continue iteration until $|f(x)| < \varepsilon$

After 7 iterations,

$x^{(7)} = 10.59$

and $|f(x^{(7)})| = 2.97 \times 10^{-6} < \varepsilon$

2. The following nonlinear equations contain terms that are often found in the power flow equations.

$f_1(\mathbf{x}) = 10\ x_1 \sin x_2 + 1.4 = 0$

$f_2(\mathbf{x}) = 10\ (x_1)^2 - 10\ x_1 \cos x_2 + 0.6\ = 0$

Using the Newton-Raphson method, determine a solution. Start with an initial guess of $x_1(0) = 1$ and $x_2(0) = 0$ radians, and a stopping criteria of $\varepsilon = 10^{-4}$.

```
-------------------------iteration 1 ---------------------------
jac= [[ 0. 10.]
 [10.  0.]]
jac inverse= [[0.  0.1]
 [0.1 0. ]]
x1= 0.94
x2= -0.13999999999999999
x= [[ 0.94]
 [-0.14]]
-------------------------iteration 2 ---------------------------
jac= [[-1.39543115  9.30803036]
 [ 8.89784004 -1.31170528]]
jac inverse= [[0.0161957  0.11492677]
 [0.10986212 0.01722947]]
x1= 0.9238628689907368
x2= -0.1519050941209388
x= [[ 0.92386287]
 [-0.15190509]]
-------------------------iteration 3 ---------------------------
jac= [[-1.51321562  9.13224209]
 [ 8.59241148 -1.39800373]]
jac inverse= [[0.01830986 0.11960633]
 [0.11253608 0.01981881]]
x1= 0.9234694225447472
x2= -0.15218888468411806
x= [[ 0.92346942]
 [-0.15218888]]
[[ 0.92346942]
 [-0.15218888]]

Process finished with exit code 0
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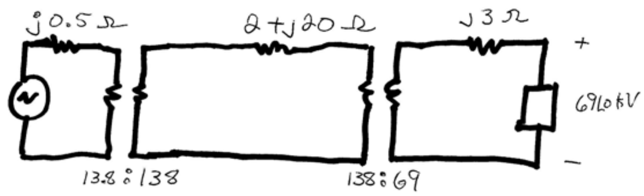
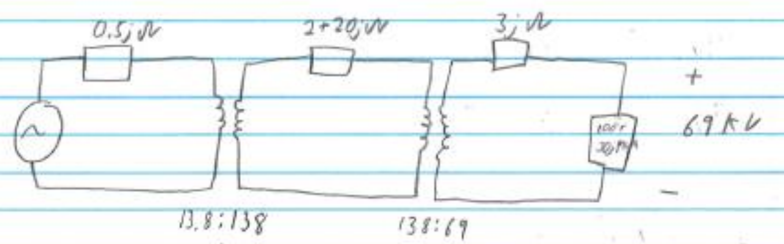
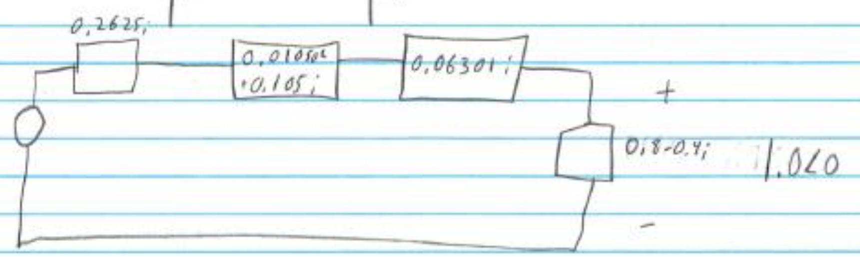
3. Assume the below diagram models a balanced three-phase system in which a 100 + j50 MVA load (total for all three phases) is supplied at 69 kV (line-to-line). First, redraw the network using a per unit representation with a 100 MVA base, and a 69 kV voltage base for the load. Then solve the circuit to determine how much real and reactive power is being supplied by the generator (source) on the left?

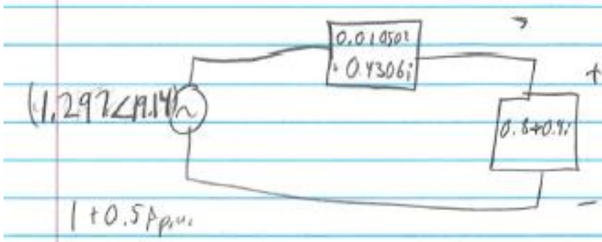


3.



$S_B = 100\ MVA$	$S_B = 100\ MVA$	$S_B = 100\ MVA$
$V_{B_1} = 13.8\ kV$	$V_{B2} = 138\ kV$	$V_{B_3} = 69\ kV$
$Z_{B_1} = \frac{13.8kV^2}{100MVA} = 1.9044\ \Omega$	$Z_{B2} = \frac{138kV^2}{100MVA} = 190.44$	$Z_{B3} = \frac{69kV^2}{100MVA} = 47.61\ \Omega$
$Z_{pu} = \frac{0.5j}{1.9044} = 0.2625j$	$Z_{pu} = \frac{2+20j}{190.44} = \frac{0.010502}{+0.105j}$	$Z_{pu} = \frac{3j}{47.61} = 0.063012j$





$P = VI$

$100 + 50j\ MVA = 69kV$

$1\angle 0\ V_{p.u.}$

$V = IR$

$1 - 0.5j\ A_{p.u.}$

$1 + 0.5\ A_{p.u.}$

$\boxed{101.31 + 103.81j\ MVA}$