ECEN 615 Problem Set #6 Fall 2018

Due 11/29/18

1. For the lossless two bus system from lecture 22, pick a value of $P_L = 4 +$ (last two digits in your UIN/100) and determine the associated value of Q_L that is on the solvability boundary Σ. First, calculate the power flow solution associated with this boundary point. Next, calculate the normal to the boundary in parameter space. Then, use the normal to change the load slightly moving into the solvable region. Determine the two power flow solutions associated with this point. Give the two solutions, and comment on how they relate to the boundary point power flow solution and then right eigenvector associated with the zero eigenvalue of the boundary point.

Solution

Let $P_L = 4+(56/100) = 4.56$, and B = -10 from the 2-bus oneline diagram
For the two bus system, corresponding maximum Q loadablity,

$$Q_L = \frac{{P_L}^2}{\mathrm{B}} - \frac{\mathrm{B}}{4} = \frac{4.56^2}{-10} + \frac{10}{4} = 0.42$$

Power flow equations for 2-bus system:

$$P_L - BV\mathrm{Sin}\theta = 0;\; Q_L + BVCos\theta - BV^2 = 0$$

Analytically solving for V and θ by susbstitution

$$BV\mathrm{Sin}\theta = P_L;\; BV\mathrm{Cos}\theta = BV^2 - Q_L$$
$$(BV\mathrm{Sin}\theta)^2 + (BV\mathrm{Cos}\theta)^2 = (P_L)^2 + (BV^2 - Q_L)^2$$
$$B^2V^2 = {P_L}^2 + B^2V^4 - 2BV^2Q_L + {Q_L}^2$$
$$B^2V^4 - (2BQ_L + B^2)V^2 + \left({P_L}^2 + {Q_L}^2\right) = 0$$

If $x = V^2$, then $B^2x^2 - (2BQ_L + B^2)x + \left({P_L}^2 + {Q_L}^2\right) = 0$. Solve for x.

$$x = \frac{(2BQ_L + B^2) \pm \sqrt{((2BQ_L + B^2)^2 - 4(B^2)\left({P_L}^2 + {Q_L}^2\right)}}{2B^2} \quad , thus\ V = \sqrt{\mathrm{x}} \qquad (*)$$

From $P_L - BV\mathrm{Sin}\theta = 0$,

$$\theta = Sin^{-1}\left(\frac{P_L}{\mathrm{BV}}\right) \qquad (**)$$

a. Hence, power flow solution at boundary (P_L, Q_L) using (*) and (**)

$$\boldsymbol{V_1\angle\theta_1 = 0.682\angle -41.96°}, \quad \boldsymbol{V_2\angle\theta_2 = 0.671\angle -42.81°}$$

Ideally, only one solution at boundary of bifurcation. Here, $V\angle\theta = V_1\angle\theta_1 \cong V_2\angle\theta_2$ (Approximation errors)

b. Normal to boundary is left eigenvector of Jacobian zero eigenvalue at boundary solution

$$J(V,\theta) = \begin{bmatrix} -BV\text{Cos}\theta & -B\text{Sin}\theta \\ -BV\text{Sin}\theta & B\text{Cos}\theta - 2BV \end{bmatrix}$$

Plugging in:

$$J = \begin{bmatrix} 5.071 & -6.69 \\ -4.56 & 6.20 \end{bmatrix}$$

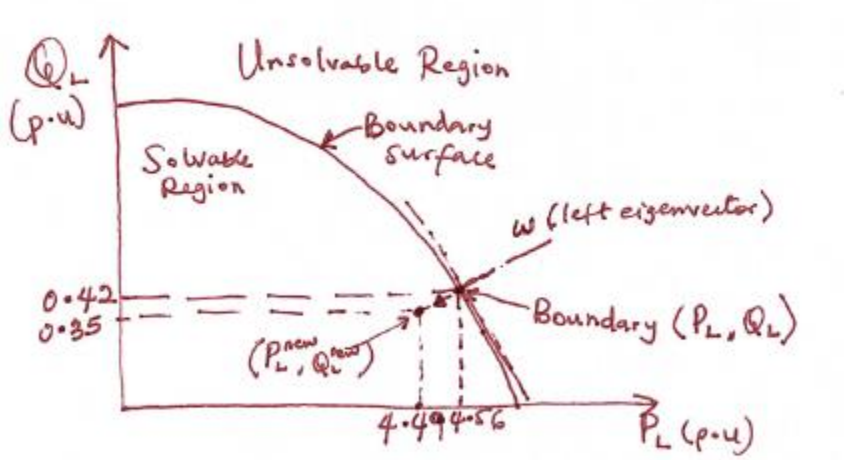
Computing left eigenvector of $\boldsymbol{J}$,

$$\boldsymbol{w} = \begin{bmatrix} -0.6749 \\ -0.7379 \end{bmatrix}$$

c. Slightly change P_L, Q_L into solvable region

$$\begin{bmatrix} {P_L}^{\boldsymbol{new}} \\ {Q_L}^{\boldsymbol{new}} \end{bmatrix} = \begin{bmatrix} P_L \\ Q_L \end{bmatrix} + \beta w$$

If $\beta = 0.1, {P_L}^{new} = 4.49, {Q_L}^{new} = 0.346$



Power flow solutions at ${P_L}^{new}, {Q_L}^{new}$ using (*) and (**)

$$\boldsymbol{V_1\angle\theta_1 = 0.59\angle-49.55°}, \quad \boldsymbol{V_2\angle\theta_2 = 0.763\angle-36.05°}$$
$$\boldsymbol{V_1\angle\theta_1 = 0.59\angle-0.865\,rad}, \quad \boldsymbol{V_2\angle\theta_2 = 0.763\angle-0.629\,rad}$$

d. Comments: The right eigenvector,$\boldsymbol{v}$ of Jacobian zero eigenvalue at boundary solution

$$\boldsymbol{v} = \begin{bmatrix} -0.8016 \\ -0.5978 \end{bmatrix}$$

$$\boldsymbol{diff} = \begin{bmatrix} \theta_{2,new} \\ V_{2,new} \end{bmatrix} - \begin{bmatrix} \theta_{1,new} \\ V_{1,new} \end{bmatrix} = \begin{bmatrix} -0.236 \\ -0.173 \end{bmatrix}$$

$\boldsymbol{diff}$ is the vector connecting high and low voltage solutions for load level ${P_L}^{new}, {Q_L}^{new}$.

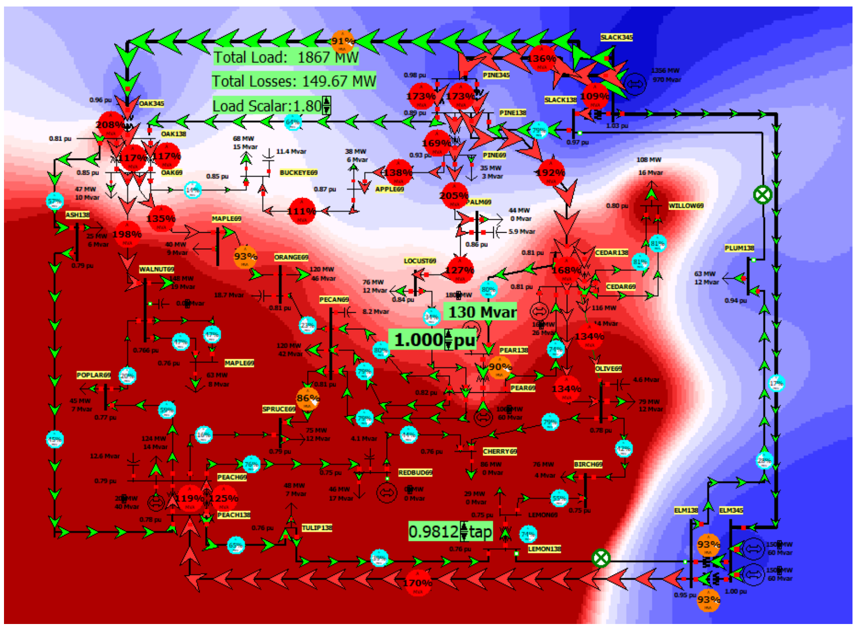
In state space, $\boldsymbol{v}$ is in the direction of voltage collapse. In a 2-bus, a direction is from the high to low voltage solution. This can be verified by checking the angle between both vectors

$$\theta = Cos^{-1}\left(\frac{\boldsymbol{diff.v}}{|\boldsymbol{diff}|\ |\boldsymbol{v}|}\right) = 0.47°$$

$\theta° \cong\ 0°$ indicates $\boldsymbol{v}$ and $\boldsymbol{diff}$ in the same direction

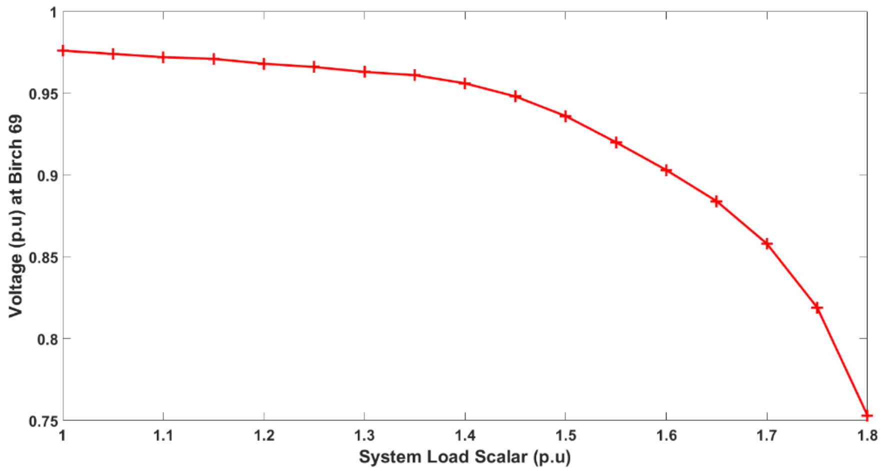
As shown already, the loading parameters for the current solution is related to the loading at the boundary point by the left eigenvector $\boldsymbol{w}$ in the parameter space. High and low voltage solutions coalesce at the boundary point.

2. In PowerWorld using the Bus37_PV system first open two transmission lines and one generator. You may choose any two lines, except with the requirement that you not isolate any load or island the system. For the generator you may open any one, excepting the slack bus generator. Then, use the **Load Scalar** field to increase the system load until the system reaches voltage collapse. Plot the PV curve, with P being the total system load, and V being the voltage magnitude at the bus that has the lowest voltage magnitude at the point of voltage collapse. Your PV curve should have at least ten fairly uniformly spaced points.



Two transmission lines opened: Slack138-Plum 138, Lemon 138-Elm138
One generator disconnected: Gen at Redbud69

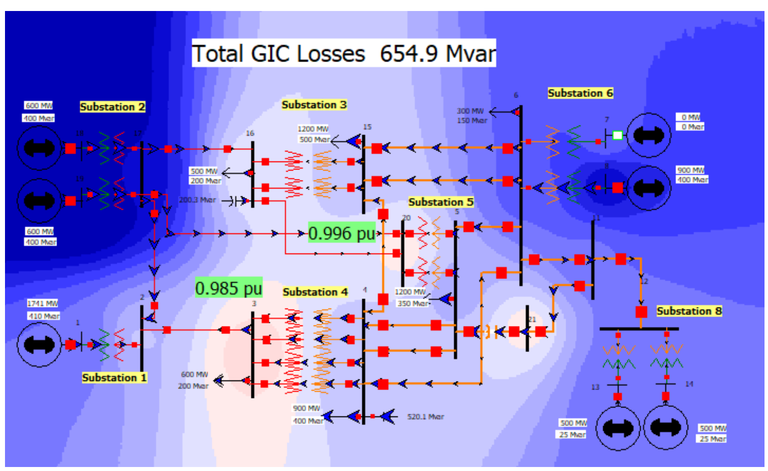
System Load Scalar	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80
V(p.u) at Birch 69	0.976	0.974	0.972	0.971	0.968	0.966	0.963	0.961	0.956	0.948	0.936	0.920	0.903	0.884	0.858	0.819	0.753



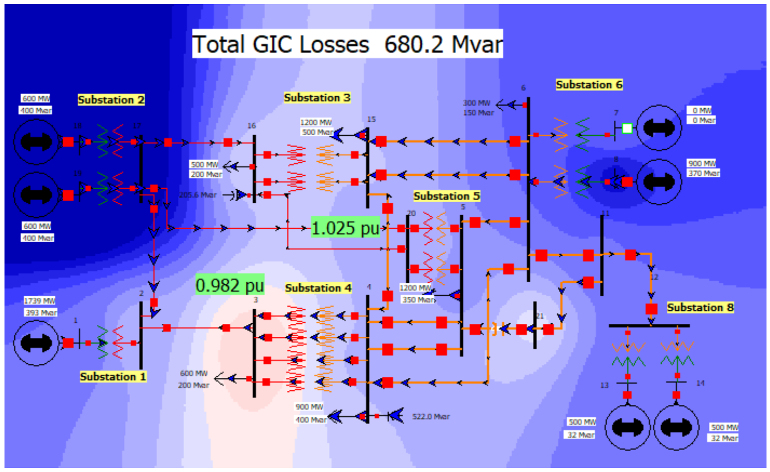
3. In PowerWorld using the GIC_20BusTestCase open one generator of your choice. Then determine the maximum GMD uniform electric field value with zero degrees before voltage collapse occurs; use a 0.5 V/km resolution. Repeat with 90 degrees.

Solution

One generator disconnected: Gen at bus 7 (substation 6)



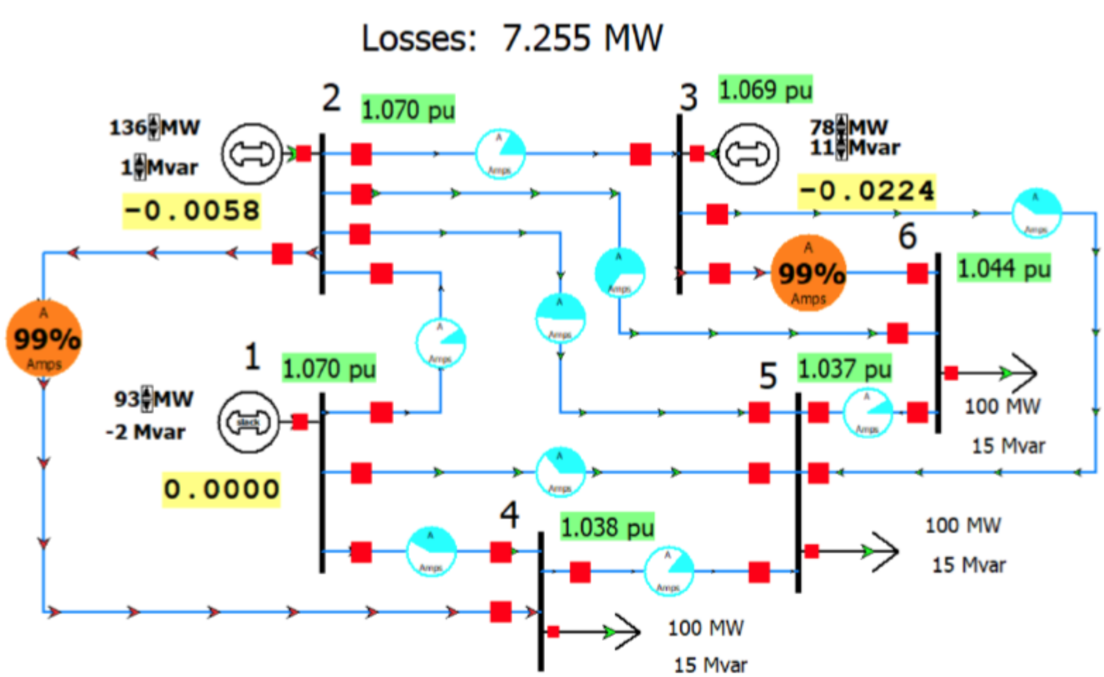
Storm direction = 0.0 deg (GIC loading prior collapse)



Storm direction: 90.0 deg (GIC loading prior collapse)

Storm Direction (deg)	0.0	90.0
Max GMD uniform fld (V/km)	4.0	3.0
Total GIC Losses (Mvar)	654.9	680.2

4. Book problem 8.3. You can do this in PowerWorld using the provided WWS_6Bus case. Note the results on page 271 are with the line between buses 1 and 4 out-of-service. You should solve it with the line in-service.



A system minimum loss of 7.255 MW (while ensuring all lines are in service and no line limit violation) is obtained when generator outputs are 136 MW (bus 2) and 78 MW (bus 3).

Method: Increase output of gen with higher loss sensitivity; Then increase output of second gen when line loading is close to limit.