Lecture 23: Geomagnetic Disturbances, Optimal Power Flow
Announcements

• Homework 6 is due on Thursday Nov 29
• Read Chapters 3 and 8 (Economic Dispatch and Optimal Power Flow)
Givens Algorithm for Factoring a Matrix $A$

- The Givens algorithm works by pre-multiplying the initial matrix, $A$, by a series of matrices and their transposes, starting with $G_1 G_1^T$
  - If $A$ is $m$ by $n$, then each $G$ is an $m$ by $m$ matrix
- Algorithm proceeds column by column, sequentially zeroing out elements in the lower triangle of $A$, starting at the bottom of each column

$$G_1 \ldots G_p G_p^T \ldots G_1^T A = QU$$
$$G_1 \ldots G_p = Q$$
$$G_p^T \ldots G_1^T A = U$$

If $A$ is sparse, then we can take advantage of sparsity going up the column
Givens Algorithm

• To zero out element $A[i,j]$, with $i > j$ we first solve with $a = A[k,j], b = A[i,j]$

$$
\begin{bmatrix}
c & s \\
-s & c
\end{bmatrix}^T
\begin{bmatrix}
a \\
b
\end{bmatrix} =
\begin{bmatrix}
r \\
0
\end{bmatrix}
$$

• A numerically safe algorithm is

If $b = 0$ then $c = 1$, $s = 0$ // i.e, no rotation is needed

Else If $|b| > |a|$ then $\tau = -a / b; \quad s = 1 / \sqrt{1 + \tau^2}; c = s\tau$

Else $\tau = -b / a; \quad c = 1 / \sqrt{1 + \tau^2}; s = c\tau$
The orthogonal $G(i,k,\theta)$ matrix is then

$$G(i,k,\theta) = \begin{bmatrix}
1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & c & \cdots & s & \cdots & 0 \\
0 & \cdots & -s & \cdots & c & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 1
\end{bmatrix}$$

Premultiplication by $G(i,k,\theta)^T$ is a rotation by $\theta$ radians in the $(i,k)$ coordinate plane.
Small Givens Example

- Let $A = \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & 5 \\ 2 & 1 \end{bmatrix}$

- First we zero out $A[4,1]$, $a=1$, $b=2$ giving $s= 0.8944$, $c=-0.4472$

First start in column $j=1$; we will zero out $A[4,1]$ with $i=4$, $k=2$
Small Givens Example

- Next zero out $A[2,1]$ with $a=4$, $b=-2.236$, giving $c=-0.8729$, $s=0.4880$

$$G_2 = \begin{bmatrix} 0.873 & 0.488 & 0 & 0 \\ -0.488 & 0.873 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G_1^T G_2^T A = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5 \\ 0 & -0.447 \end{bmatrix}$$

- Next zero out $A[4,2]$ with $a=5$, $b=-0.447$, $c=0.996$, $s=0.089$

$$G_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.996 & 0.089 \\ 0 & 0 & -0.089 & 0.996 \end{bmatrix} \quad G_1^T G_2^T G_3^T A = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5.020 \\ 0 & 0 \end{bmatrix}$$
Small Givens Example

- Next zero out $A[3,2]$ with $a=0.195$, $b=5.02$, $c=0.996$, $s=0.089$

$$G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.039 & 0.999 & 0 \\ 0 & -0.999 & -0.039 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G_4^T G_3^T G_2^T G_1^T A = \begin{bmatrix} 4.58 & 2.18 \\ 0 & -5.023 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Also we have

$$Q = G_1 G_2 G_3 G_4 = \begin{bmatrix} 0.872 & -0.019 & 0.487 & 0 \\ 0.218 & 0.094 & -0.387 & 0.891 \\ 0 & -0.995 & -0.039 & 0.089 \\ 0.436 & -0.009 & -0.782 & -0.445 \end{bmatrix}$$
Back to GMDs: Solar Cycles

- Sunspots follow an 11 year cycle, and have been observed for hundreds of years.
- We're in solar cycle 24 (first numbered cycle was in 1755); minimum was in 2009, maximum in 2014/2015.

Images from NASA, NOAA
But Large CMEs Are Not Well Correlated with Sunspot Maximums

The large 1921 storm occurred four years after the 1917 maximum.
In July 2014 NASA said in July of 2012 there was a solar CME that barely missed the earth – It would likely have caused the largest GMD that we have seen in the last 150 years

There is still lots of uncertainty about how large a storm is reasonable to consider in electric utility planning

Image Source: science.nasa.gov/science-news/science-at-nasa/2014/23jul_superstorm/
The two key concerns from a big storm are 1) large-scale blackout due to voltage collapse, 2) permanent transformer damage due to overheating.
Geomagnetically Induced Currents (GICs)

- GMDs cause slowly varying electric fields
- Along length of a high voltage transmission line, electric fields can be modeled as a dc voltage source superimposed on the lines
- These voltage sources produce quasi-dc geomagnetically induced currents (GICs) that are superimposed on the ac (60 Hz) flows
GIC Calculations for Large Systems

- With knowledge of the pertinent transmission system parameters and the GMD-induced line voltages, the dc bus voltages and flows are found by solving a linear equation \( \mathbf{I} = \mathbf{G} \mathbf{V} \) (or \( \mathbf{J} = \mathbf{G} \mathbf{U} \))
  - \( \mathbf{J} \) and \( \mathbf{U} \) may be used to emphasize these are dc values, not the power flow ac values
  - The \( \mathbf{G} \) matrix is similar to the \( \mathbf{Y}_{\text{bus}} \) except 1) it is augmented to include substation neutrals, and 2) it is just resistive values (conductances)
    - Only depends on resistance, which varies with temperature
  - Being a linear equation, superposition holds
  - The current vector contains the Norton injections associated with the GMD-induced line voltages
GIC Calculations for Large Systems

- Factoring the sparse $G$ matrix and doing the forward/backward substitution takes about 1 second for the 60,000 bus Eastern Interconnect Model.

- The current vector ($I$) depends upon the assumed electric field along each transmission line.
  - This requires that substations have correct geo-coordinates.

- With nonuniform fields an exact calculation would be path dependent, but just assuming a straight line path is probably sufficient (given all the other uncertainties!)
Four Bus Example (East-West Field)

\[ I_{GIC,3\text{Phase}} = \frac{150 \text{ volts}}{1 + 0.1 + 0.1 + 0.2 + 0.2} \Omega = 93.75 \text{ amps or 31.25 amps/phase} \]

Substation A with \( R = 0.2 \) ohm
- Neutral = 18.7 Volts
- Bus 3:
  - DC = 18.7 Volts
  - 1.001 pu
- Bus 1:
  - DC = 28.1 Volts
  - 0.999 pu

Substation B with \( R = 0.2 \) ohm
- Neutral = -18.7 Volts
- Bus 2:
  - DC = -28.1 Volts
  - 0.997 pu
- Bus 4:
  - DC = -18.7 Volts
  - 1.000 pu

765 kV Line
- 3 ohms Per Phase
- High Side = 0.3 ohms/Phase
- GIC/Phase = 31.2 Amps

GIC Losses = 25.5 Mvar

The line and transformer resistance and current values are per phase so the total current is three times this value. Substation grounding values are total resistance. Brown arrows show GIC flow.

Case name is GIC_FourBus
Four Bus Example GIC G Matrix

\[ \mathbf{U} = [\mathbf{G}]^{-1} \mathbf{J} \]

\[
\begin{bmatrix}
18.75 \\
-18.75 \\
28.12 \\
-28.12
\end{bmatrix} =
\begin{bmatrix}
15 & 0 & -10 & 0 \\
0 & 15 & 0 & -10 \\
-10 & 0 & 11 & -1 \\
0 & -10 & -1 & 11
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
0 \\
150 \\
-150
\end{bmatrix}
\]
GICs, Generic EI, 5 V/km East-West
GICs, Generic EI, 5 V/km North-South
Determining GMD Storm Scenarios

• The starting point for the GIC analysis is an assumed storm scenario; sets the line dc voltages
• Matching an actual storm can be complicated, and requires detailed knowledge of the geology
• GICs vary linearly with the assumed electric field magnitudes and reactive power impacts on the transformers is also mostly linear
• Working with space weather community to determine highest possible storms
• NERC proposed a non-uniform field magnitude model that FERC has partially accepted
Electric Field Linearity

- If an electric field is assumed to have a uniform direction everywhere (like with the current NERC model), then the calculation of the GICs is linear
  - The magnitude can be spatially varying
- This allows for very fast computation of the impact of time-varying functions (like with the NERC event)
- PowerWorld now provides support for loading a specified time-varying sequence, and quickly calculating all of the GIC values
The two key concerns from a big storm are 1) large-scale blackout due to voltage collapse, 2) permanent transformer damage due to overheating.
Impact of Earth Models: Relationship Between dB/dT and E

- The magnitude of the induced electric field depends upon the rate of change in the magnetic field, and the deep earth (potentially 100’s of km) conductivity

- The relationship between changing magnetic fields and electric fields are given by the Maxwell-Faraday Equation

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  

\[ \oint_{\partial \Sigma} \mathbf{E} \cdot d\ell = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot dS \]  

Faraday's law is \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \)  
Faraday's law is \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \)
Relationship Between dB/dT and E

• If the earth is assumed to have a single conductance, $\sigma$, then

$$ Z(\omega) = \frac{j\omega\mu_0}{\sqrt{j\omega\mu_0\sigma}} = \sqrt{\frac{j\omega\mu_0}{\sigma}} $$

• The magnitude relationship is then

Recalling $B(\omega) = -\mu_0 H(\omega)$

$$ |E(\omega)| = |Z(\omega) H(\omega)| $$

$$ = \left| \sqrt{\frac{j\omega\mu_0}{\sigma}} \frac{B(\omega)}{\mu_0} \right| $$

For example, assume $\sigma$ of 0.001 S/m and a 500nT/minute maximum variation at 0.002 Hz. Then

$$ B(\omega) = 660 \times 10^{-9} \text{ T} \text{ and} $$

$$ E(\omega) = \sqrt{\frac{2\pi \times 0.002 \times \mu_0}{0.001 \mu_0}} \frac{660 \times 10^{-9}}{\mu_0} \text{ T} $$

$$ E(\omega) = 0.00397 \times 0.525 = 2.1 \text{ V/km} $$

A more resistive earth gives higher electric fields
Typical Conductance and Resistivity Values

- Soil conductance is often expressed in its inverse of resistivity in $\Omega\cdot m$; values can vary widely
  - Topsoil varies widely with moisture content, from 2500 $\Omega\cdot m$ when dry to about 20 $\Omega\cdot m$ when very wet
  - Clay is between 100-200 $\Omega\cdot m$

Image source: https://www.eoas.ubc.ca/courses/eosc350/content/foundations/properties/resistivity.htm
1-D Earth Models

- With a 1-D model the earth is model as a series of conductivity layers of varying thickness.

- The impedance at a particular frequency is calculated using a recursive approach, starting at the bottom, with each layer $m$ having a propagation constant

$$k_m = \sqrt{j\omega \mu_0 \sigma_m}$$

- At the bottom level $n$:

$$Z_n = \frac{j\omega \mu_0}{k_n}$$
1-D Earth Models

- Above the bottom layer each layer $m$, has a reflection coefficient associated with the layer below

$$r_m = \frac{(1 - k_m) \frac{Z_{m+1}}{j\omega\mu_0}}{(1 + k_m) \frac{Z_{m+1}}{j\omega\mu_0}}$$

- With the impedance at the top of layer $m$ given as

$$Z_m = j\omega\mu_0 \left( \frac{1 - r_m e^{-2k_md_m}}{k_m \left(1 + r_m e^{-2k_md_m}\right)} \right)$$

- Recursion is applied up to the surface layer
The USGS has broken the continental US into about 20 conductivity (resistivity) regions. These regional scalings are now being used for power flow GMD analysis, and are being updated.

Image from the NERC report; data is available at http://geomag.usgs.gov/conductivity/
1-D Earth Models

- Image on the bottom left shows an example 1-D model, whereas image on bottom right shows the $Z(\omega)$ variation for two models.
The magnetotelluric (MT) component of USArray, an NSF Earthscope project, consists of 7 permanent MT stations and a mobile array of 20 MT stations that will each be deployed for a period of about one month in regions of identified interest with a spacing of approximately 70 km. These MT measurements consist of magnetic and electric field data that can be used to calculate 3D conductivity deep in the Earth. The MT stations are maintained by Oregon State University’s National Geoelectromagnetic Facility, PI Adam Schultz. (www.earthscope.org)
3-D Models and EarthScope

- EarthScope data is processed into magnetotelluric transfer functions that:
  - Define the frequency dependent linear relationship between EM components at a single site.
    \[
    \frac{E_x(\omega)}{B_y(\omega)} = \xi_{xy}
    \]
    (simplified for the 1D case)
  - Can be used to relate a magnetic field input to and electric field output at a single site
    \[
    \begin{bmatrix}
    E_x \\
    E_y
    \end{bmatrix} =
    \begin{bmatrix}
    \xi_{xx} & \xi_{xy} \\
    \xi_{yx} & \xi_{yy}
    \end{bmatrix}
    \cdot
    \begin{bmatrix}
    B_x \\
    B_y
    \end{bmatrix}
    \]
  - Are provided in 2x2 impedance tensors by USArray

Reference: Kelbert et al., IRIS DMC Data Services Products, 2011.
Example 3-D Earthscope Model Results

- Image provides a snapshot visualization of the time-varying surface electric fields using Earthscope data

White ~ 10 V/km
Image Provided by Jenn Gannon
Input Electric Field Considerations

• The current vector (I) depends upon the assumed electric field along each transmission line

• With a uniform electric field determination of the transmission line’s GMD-induced voltage is path independent
  – Just requires geographic knowledge of the transmission line’s terminal substations

• With nonuniform fields an exact calculation would be path dependent, but just a assuming a straight line path is probably sufficient (given all the other uncertainties!)
The two key concerns from a big storm are 1) large-scale blackout due to voltage collapse, 2) permanent transformer damage due to overheating.
Transformer Impacts of GICs

- The GICs superimpose on the ac current, causing transformers saturation for part of the ac cycle.
- This can cause large harmonics; in the positive sequence these harmonics can be represented by increased reactive power losses in the transformer.

Images: Craig Stiegemeier and Ed Schweitzer, JASON Presentations, June 2011
Relating GICs to Transformer Mvar Losses

- Transformer positive sequence reactive power losses vary as a function of both the GICs in the transformer coils and the ac voltage.
- A common approach is to use a linear model:

\[ Q_{loss} = KV_{pu} I_{GIC,\text{Eff}} \]

- The \( I_{GIC,\text{Eff}} \) is an effective current that is a function of the GICs in both coils; whether auto or regular the equation is:

\[ I_{GIC,\text{Eff}} = \left| \frac{a_t I_{GIC,H} + I_{GIC,L}}{a_t} \right| \]

where \( a_t \) is the turns ratio.
Confusions!

- $I_{GIC,\text{Eff}}$ is a dc value, whereas $Q_{\text{loss}}$ is the total ac losses.
- From a GIC perspective, the three phases are in parallel; hence there can be confusion as to whether the GICs are per phase or total (per phase is common and used in PowerWorld).
- Since $Q_{\text{loss}}$ varies linearly with voltage, the nominal high voltage of the transformer (e.g., 500 kV, 345 kV, etc.) needs to be embedded in the K.
  - An initial approach was to assume a K for 500 kV; the K then needed to be scaled for other voltages.
Specifying Transformer Losses Scalars in Per Unit

- Alternative approach (used here) of representing the $K$ values in per unit
  - Using a base derived from transformer’s high side voltage
  - Current base using the peak value

$$I_{\text{base,highkv,peak}} = \frac{S_{\text{base,xf}} 1000\sqrt{2}}{V_{\text{base,highkv}} \sqrt{3}}$$

- Convert to per unit by dividing $Q_{\text{loss}}$ by $S_{\text{base,xf}}$

$$\frac{Q_{\text{loss}}}{S_{\text{base,xf}}} = \frac{V_{\text{pu}} K_{\text{old}}}{500} \frac{V_{\text{base,highkv}}}{1000 I_{\text{GIC,Eff}} \sqrt{2}} \frac{V_{\text{base,highkv}} I_{\text{base,highkv,peak}}}{\sqrt{3}}$$

$$Q_{\text{loss,pu}} = \frac{1000\sqrt{2} K_{\text{old}}}{500\sqrt{3}} I_{\text{eff,pu}} = V_{\text{pu}} \left(1.633 K_{\text{old}}\right) I_{\text{eff,pu}} = V_{\text{pu}} K_{\text{new}} I_{\text{eff,pu}}$$

$K_{\text{new}} = 1.633 K_{\text{old}}$

K$_{\text{old}}$ based on an assumed 500 kV

Peak (or “crest” value used since this is a dc current base
The two key concerns from a big storm are 1) large-scale blackout due to voltage collapse, 2) permanent transformer damage due to overheating.
GMD Enhanced Power Analysis Software

- By integrating GIC calculations directly within power flow and transient stability engineers can see the impact of GICs on their systems, and consider mitigation options
- GIC calculations use many of the existing model parameters such as line resistance. Some non-standard values are also needed; either provided or estimated
  - Substation grounding resistance
  - transformer grounding configuration
  - transformer coil resistance
  - whether auto-transformer, three-winding transformer
Small 20 Bus Benchmark System Example

Contour shows the ac, per unit bus voltage magnitudes

Basecase loading is 4700 MW and 1800 Mvar. Transmission voltages are 345 and 500 kV.
Assumed Geographic Location (Mostly East-West)
GIC Flows with a 1V/km North-South, Uniform Electric Field

Total GIC Losses  528.9 Mvar

Substation 1
- 600 MW
- 121 Mvar
- 200 Mvar
- 779 MW
- 100 Mvar

Substation 2
- 600 MW
- 121 Mvar

Substation 3
- 1200 MW
- 500 Mvar
- 204.3 Mvar

Substation 4
- 600 MW
- 200 Mvar
- 900 MW
- 400 Mvar

Substation 5
- 1200 MW
- 350 Mvar

Substation 6
- 900 MW
- 55 Mvar

Substation 8
- 500 MW
- 17 Mvar

Total GIC Losses  528.9 Mvar

Voltage\Per Unit
-1.100 pu
-1.000 pu
-0.900 pu

Flow Diagram
GIC Flows with a 1V/km East-West, Uniform Electric Field

The GIC losses are higher with an east-west field since the system mostly goes in that direction.
GIC Flows with a 2V/km East-West, Uniform Electric Field

Total GIC Losses 1474.9 Mvar
GIC Flows with a 2.2V/km East-West, Uniform Electric Field – By Voltage Collapse

Total GIC Losses 1587.5 Mvar
The two key concerns from a big storm are 1) large-scale blackout due to voltage collapse, 2) permanent transformer damage due to overheating.
NERC GMD Scenario Approach

- Time varying, derived from the March 1989 event
- Peak electric field is 8 V/km for a reference location (60 deg. N, resistive Earth)
- Electric field for other regions scaled by two factors
  - $E_{\text{peak}} = 8 \times \alpha \times \beta \text{ V/km}$
  - “1 in a 100 year” event

The beta scalars depend on the assumed 1-D models
Geomagnetic Latitude
The Impact of a Large GMD From an Operations Perspective

• Maybe a day warning but without specifics
  – Satellite at Lagrange point one million miles from earth would give more details, but with less than 30 minutes lead time
  – Could strike quickly; rise time of minutes, rapidly covering a good chunk of the continent
• Reactive power loadings on hundreds of high voltage transformers could rapidly rise
The Impact of a Large GMD From an Operations Perspective

- Increased transformer reactive loading causes heating issues and potential large-scale voltage collapses
- Power system software like state estimation could fail
- Control room personnel would be overwhelmed
- The storm could last for days with varying intensity
- Waiting until it occurs to prepare would not be a good idea!