ECEN 615
Methods of Electric Power Systems Analysis

Lecture 19: State Estimation

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

Announcements

- Homework 4 is due today
- Homework 5 is due on Tuesday Nov 13
- Final exam is Wednesday Dec 12, 1 to 3pm (the syllabus had indicated Wednesday Dec 13)

Example: Two Bus Case

- Assume a two bus case with a generator supplying a load through a single line with x=0.1 pu. Assume measurements of the p/q flow on both ends of the line (into line positive), and the voltage magnitude at both the generator and the load end. So $B_{12} = B_{21} = 10.0$

$$P_{ij}^{meas} - \left[V_i V_j \left(B_{ij} \sin\left(\theta_i - \theta_j \right) \right) \right]$$

$$Q_{ij}^{meas} - \left[V_i^2 B_{ij} + V_i V_j \left(-B_{ij} \cos\left(\theta_i - \theta_j \right) \right) \right]$$

$$V_i^{meas} - V_i = 0$$

We need to assume a reference angle unless we directly measuring phase

Example: Two Bus Case

- Let

$$\mathbf{Z}^{meas} = \begin{bmatrix} P_{12} \\ Q_{12} \\ P_{21} \\ Q_{21} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 1.01 \\ 0.87 \end{bmatrix} \qquad x^0 = \begin{bmatrix} V_1 \\ \theta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \sigma_i = 0.01$$

We assume an angle reference of $\theta_1 = 0$

$$H(\mathbf{x}) = \begin{bmatrix} V_2 10 \sin(-\theta_2) & -V_1 V_2 10 \cos(-\theta_2) & V_1 10 \sin(-\theta_2) \\ 20V_1 - V_2 10 \cos(-\theta_2) & -V_1 V_2 10 \sin(-\theta_2) & -V_1 10 \cos(-\theta_2) \\ V_2 10 \sin(\theta_2) & V_1 V_2 10 \cos(\theta_2) & V_1 10 \sin(\theta_2) \\ -V_2 10 \cos(\theta_2) & V_1 V_2 10 \sin(\theta_2) & 20V_2 - V_1 10 \cos(\theta_2) \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Two Bus Case

- With a flat start guess we get

$$H(\mathbf{x}^0) = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{z} - \mathbf{f}(\mathbf{x}^0) = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

Example: Two Bus Case

$$\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = 1e^6 \times \begin{bmatrix} 2.01 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 2.01 \end{bmatrix}$$

$$\mathbf{x}^1 = \mathbf{x}^0 + \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix} = \begin{bmatrix} 1.003 \\ -0.2 \\ 0.8775 \end{bmatrix}$$

Assumed SE Measurement Accuracy

- The assumed measurement standard deviations can have a significant impact on the resultant solution, or even whether the SE converges
- The assumption is a Gaussian (normal) distribution of the error with no bias

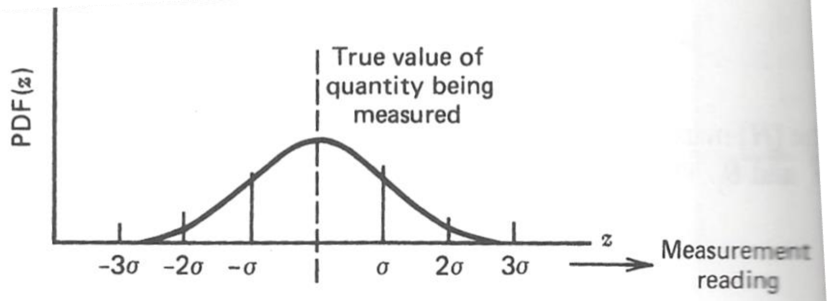


FIGURE 9.8 Normal distribution of meter errors.

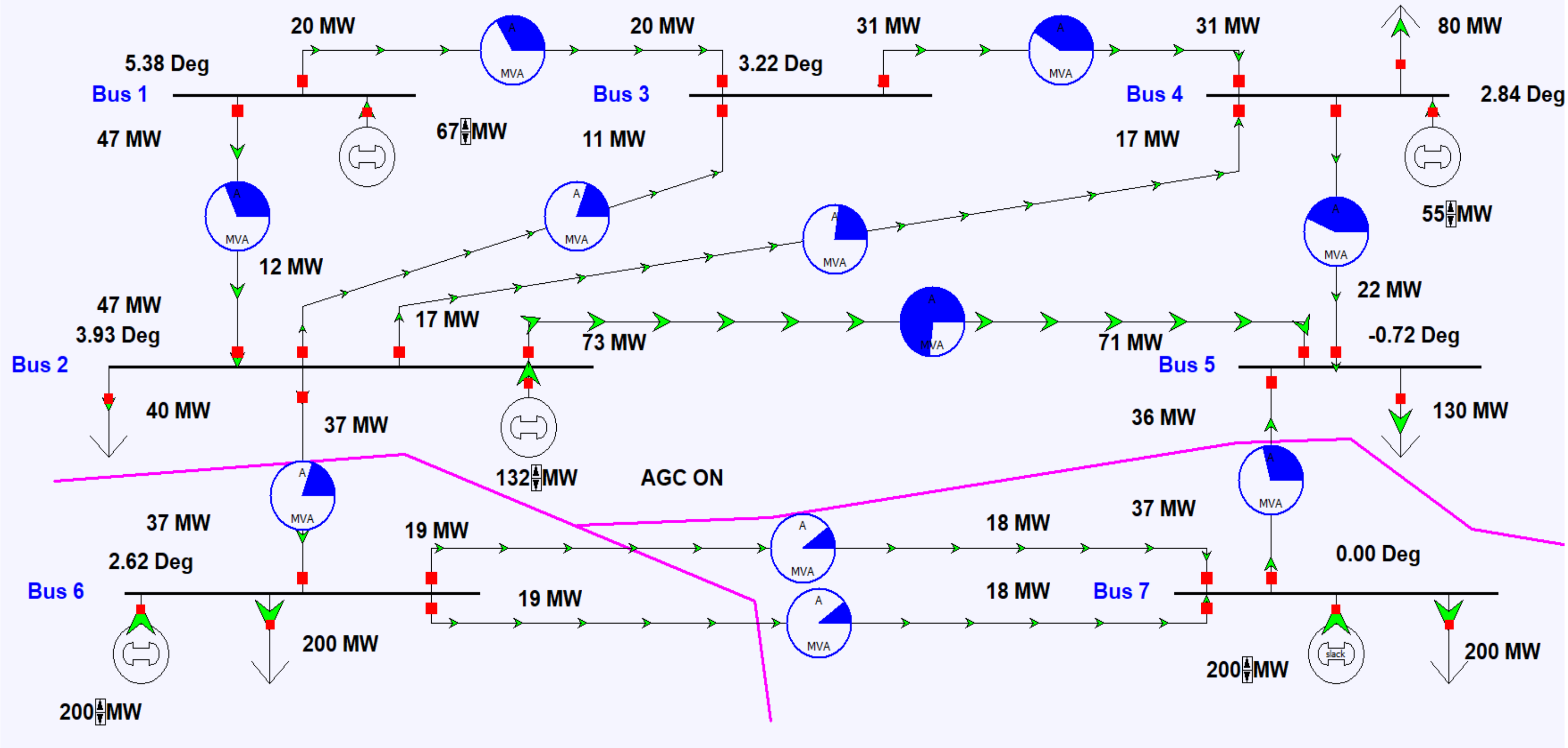
SE Observability

- In order to estimate all n states we need at least n measurements. However, where the measurements are located is also important, a topic known as observability
  - In order for a power system to be fully observable usually we need to have a measurement available no more than one bus away
  - At buses we need to have at least measurements on all the injections into the bus except one (including loads and gens)
  - Loads are usually flows on feeders, or the flow into a transmission to distribution transformer
  - Generators are usually just injections from the GSU

Pseudo Measurements

- Pseudo measurements are used at buses in which there is no load or generation; that is, the net injection into the bus is know with high accuracy to be zero
  - In order to enforce the net power balance at a bus we need to include an explicit net injection measurement
- To increase observability sometimes estimated values are used for loads, shunts and generator outputs
  - These “measurements” are represented as having a higher much standard deviation

SE Observability Example



SE Bad Data Detection

- The quality of the measurements available to an SE can vary widely, and sometimes the SE model itself is wrong. Causes include
    - Modeling Errors: perhaps the assumed system topology is incorrect, or the assumed parameters for a transmission line or transformer could be wrong
    - Data Errors: measurements may be incorrect because of in correct data specifications, like the CT ratios or even flipped positive and negative directions
    - Transducer Errors: the transductors may be failing or may have bias errors
    - Sampling Errors: SCADA does not read all values simultaneously and power systems are dynamic

SE Bad Data Detection

- The challenge for SE is to determine when there is likely a bad measurement (or multiple ones), and then to determine the particular bad measurements
- J(**x**) is random number, with a probability density function (PDF) known as a chi-squared distribution, χ^2(K), where K is the degrees of freedom, K=m-n
- It can be shown the expected mean for J(**x**) is K, with a standard deviation of $\sqrt{2K}$
  - Values of J(**x**) outside of several standard deviations indicate possible bad measurements, with the measurement residuals used to track down the likely bad measurements
- SE can be re-run without the bad measurements

Example SE Application: PJM and MISO

- PJM provides information about their EMS model in
  - www.pjm.com/-/media/documents/manuals/m03a.ashx

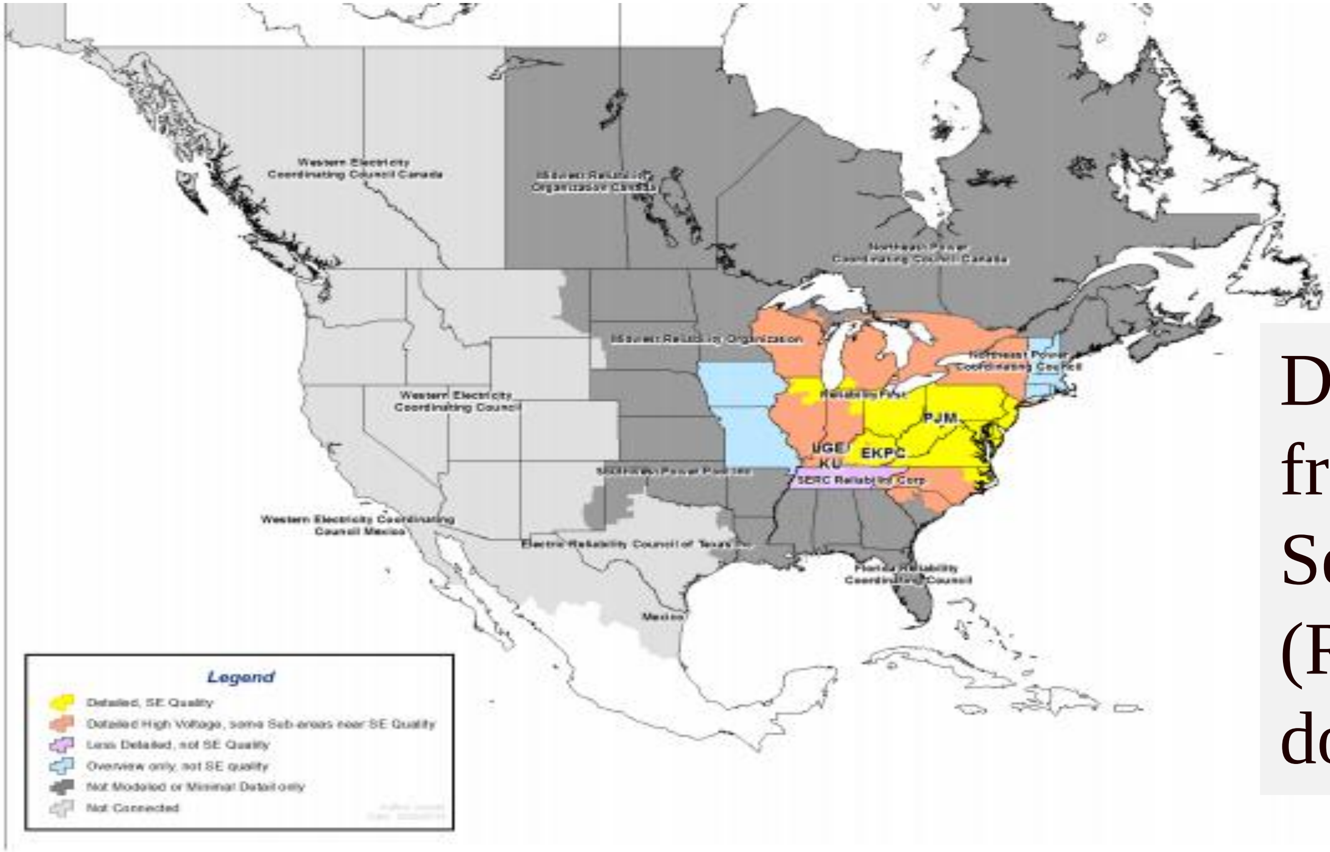


Exhibit 4: PJM EMS Model Details

Data here is from the Sept 2018 (Rev 16) document

Example SE Application: PJM and MISO

- PJM measurements are required for 69 kV and up
- PJM SE is triggered to execute every minute
- PJM SE solves well over 98% of the time
- Below reference provides info on MISO SE from March 2015
  - 54,433 buses
  - 54,415 network branches
  - 6332 generating units
  - 228,673 circuit breakers
  - 289,491 mapped points

https://www.naspi.org/sites/default/files/2017-05/3a%20MISO-NASPIWokshop-Synchrophasor%20Data%20and%20State%20Estimation.pdf