ECEN 615 Problem Set #1 Fall 2018

Due 9/13/18

1. Use Newton-Raphson to find one solution to the polynomial equation $f(x) = x^3 - 9x^2 - 14x - 30 = 0$. Start with an intial guess of 0 and continue until the mismatch is below a tolerance of $\varepsilon = 0.001$.

NR update method:

$$x^{(new)} = x^{(old)} - \frac{f(x^{(old)})}{f'(x^{(old)})}$$

$f(x) = x^3 - 9x^2 - 14x - 30$

$f'(x) = 3x^2 - 18x - 14$

$x^{ini} = 0$, $\mathcal{E} = 0.001$ (Mismatch)

First iteration:

$x^{(0)} = x^{ini} = 0$

$f(x^{(0)}) = -30$

$f'(x^{(0)}) = -14$

$$x^{(1)} = x^{(0)} - \frac{f(x^{(0)})}{f'(x^{(0)})} = 0 - \left(\frac{-30}{-14}\right) = -2.143$$

Check: $f(x^{(1)}) = (-2.143)^3 - 9(-2.143)^2 - 14(-2.143) - 30$

$= -51.17$

$|f(x^{(1)})| = 51.17 > \mathcal{E}$, hence do Second iteration

Second iteration

$$x^{(2)} = x^{(1)} - \frac{f(x^{(1)})}{f'(x^{(1)})}$$

$f(x^{(1)}) = -51.17$

$f'(x^{(1)}) = +38.35$

$$x^{(2)} = -2.143 - \left(\frac{-51.17}{+38.35}\right)$$

$= \cancel{-2.436}\ -0.809$

Check: $f(x^{(2)}) = (-0.809)^3 - 9(-0.809)^2 - 14(-0.809) - 30$

$= -25.09$

$|f(x^{(2)})| = 25.09 > \mathcal{E}$, hence do third iteration

Continue iteration until $|f(x)| < \mathcal{E}$

After 7 iterations,

$x^{(7)} = 10.59$

and $|f(x^{(7)})| = 2.97 \times 10^{-6} < \mathcal{E}$

2. The following nonlinear equations contain terms that are often found in the power flow equations:

$f_1(\mathbf{x}) = 12\, x_1 \sin x_2 + 1.5 = 0$

$f_2(\mathbf{x}) = 12\, (x_1)^2 - 12\, x_1 \cos x_2 + 0.75 = 0$

Start with an initial guess of $x_1(0) = 1$ and $x_2(0) = 0$ radians, and a stopping criteria of $\varepsilon = 10^{-4}$.

$$\bar{x}^{ini} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \varepsilon \text{ (Mismatch)} = 10^{-4}$$

NR update

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{(new)} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{old} - \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1}^{(old)} & \frac{\partial f_1}{\partial x_2}^{(old)} \\ \frac{\partial f_2}{\partial x_1}^{(old)} & \frac{\partial f_2}{\partial x_2}^{(old)} \end{bmatrix}^{-1}}_{[J(x)]^{-1}} \underbrace{\begin{bmatrix} f_1(\bar{x}^{(old)}) \\ f_2(\bar{x}^{(old)}) \end{bmatrix}}_{[f(x)]} ; \quad \bar{x} = [x_1, x_2]$$

$$\frac{\partial f_1}{\partial x_1} = 12 \sin x_2 \; ; \; \frac{\partial f_1}{\partial x_2} = 12 x_1 \cos x_2$$

$$\frac{\partial f_2}{\partial x_1} = 24 x_1 - 12 \cos x_2 \; ; \; \frac{\partial f_2}{\partial x_2} = 12 x_1 \sin x_2$$

First iteration:

Compute entries for Jacobian, $[J(x)]$ and mismatch

$$\frac{\partial f_1}{\partial x_1}^{(old)} = 0 \; , \; \frac{\partial f_1}{\partial x_2}^{(old)} = 12$$

$$\frac{\partial f_2}{\partial x_1}^{(old)} = 12 \; ; \; \frac{\partial f_2}{\partial x_2}^{(old)} = 0$$

$$f_1(\bar{x}^{(old)}) = 1.5 \, , \; f_2(\bar{x}^{(old)}) = 0.75$$

Hence,
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 12 \\ 12 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1.5 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0.9375 \\ -0.125 \end{bmatrix}$$

Check:
$$\begin{bmatrix} f_1(\bar{x}^{(1)}) \\ f_2(\bar{x}^{(1)}) \end{bmatrix} = \begin{bmatrix} 12(0.9375)\sin(-0.125) + 1.5 \\ 12(0.9375)^2 - 12(0.9375)\cos(-0.125) + 0.75 \end{bmatrix} = \begin{bmatrix} 0.097 \\ 0.135 \end{bmatrix}$$

$f_1(\bar{x}^{(1)}) = 0.097 > 10^{-4}$

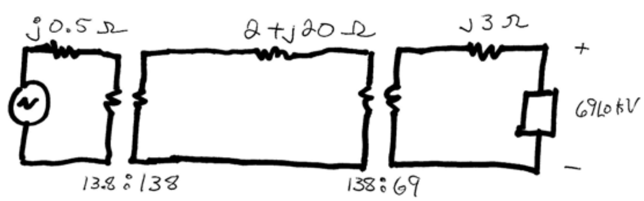
and

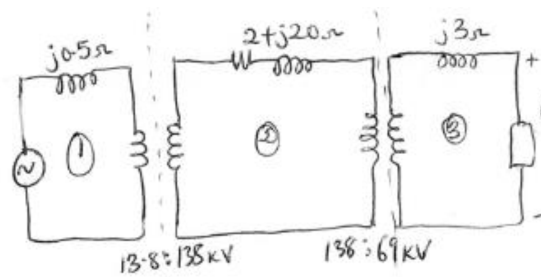
$f_2(\bar{x}^{(1)}) = 0.135 > 10^{-4}$

Hence continue with second iteration

After 4 iterations, arrive at $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{(4)} = \begin{bmatrix} 0.923 \\ -0.136 \end{bmatrix}$, and $f_1(\bar{x}^{(4)}), f_2(\bar{x}^{(4)}) < 10^{-4}$

3. Assume the below diagram models a balanced three-phase system in which a 120- + j60 MVA load (total for all three phases) is supplied at 69 kV (line-to-line). First, redraw the network using a per unit representation with a 100 MVA base, and a 69 kV voltage base for the load. How much real and reactive power is being supplied by the generator (source) on the left?





$S_L = 120 + j60$ MVA

$S_B = 100$ MVA

$Z_B = \dfrac{V_B^2}{S_B}$ (Here, V_B is in kV)

$V_{B1} = V_{B2} \times \dfrac{13.8}{138} = 13.8$ kV

$Z_{B1} = \dfrac{V_{B1}^2}{S_B} = 1.9\,\Omega$

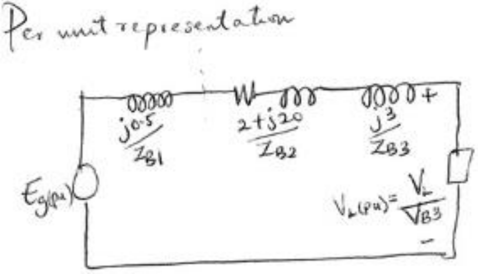
$V_{B2} = V_{B3} \times \dfrac{138}{69} = 138$ kV

$Z_{B2} = \dfrac{V_{B2}^2}{S_B} = 190.4\,\Omega$

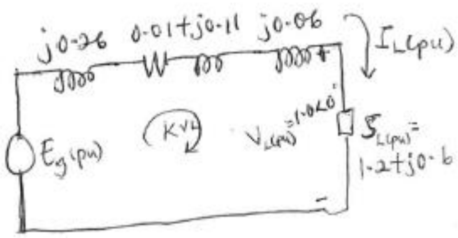
$V_{B3} = 69$ kV

$Z_{B3} = \dfrac{V_{B3}^2}{S_B} = \dfrac{69^2}{100} = 47.6\,\Omega$

Per unit representation



$\dfrac{S_L}{S_B}$ ⇒



From load,

$$S_L(pu) = V_L(pu) \times I_L^*(pu)$$

$$I_L(pu) = \left(\frac{S_L(pu)}{V_L(pu)}\right)^* = \left(\frac{1.2 + j0.6}{1.0\angle 0^\circ}\right)^* = 1.2 - j0.6\,pu$$

Using KVL,

$$E_g(pu) = V_L(pu) + I_L(pu)\,(j0.26 + j0.01 + j0.11 + j0.06)$$

$$= 1.0\angle 0^\circ + (1.2 - j0.6)(0.01 + j0.43)$$

$$= 1.27 + j0.51$$

Power generated from source

$$S_g(pu) = E_g(pu)\, I_L^*(pu) = (1.27 + j0.51)(1.2 - j0.6)^*$$

$$= 1.218 + j1.374\,pu$$

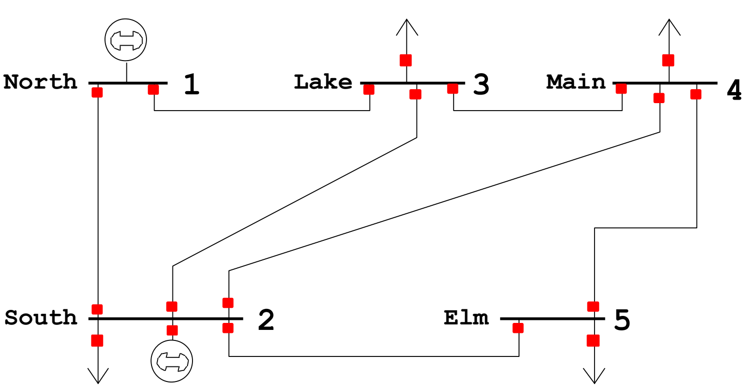
$$S_g(pu) = P_g(pu) + jQ_g(pu) \equiv 1.218 + j1.374\,pu$$

Hence, real $P_g(pu) = 1.218$ p.u. OR 121.8 MW

reactive $Q_g(pu) = 1.374$ pu. OR 137.4 Mvar

4. In the space on the next page determine the bus admittance matrix (Y_{bus}) for the following power system (note that some of the values have already been determined for you). Except where noted otherwise, assume all values are per unit using a 100 MVA base.

Sample System



Line impedances and charging values

From/to buses	Impedance Z_{ij}	Line Charging Y/2
1-2	0.02 + j 0.06	j 0.030
1-3	0.08 + j 0.24	j 0.025
2-3	0.06 + j 0.18	j 0.020
2-4	0.08 + j 0.24	j 0.025
2-5	0.02 + j 0.06	j 0.010
3-4	0.04 + j 0.12	j 0.015
4-5	0.06 + j 0.18	j 0.020

$Y_{12} = Y_{12} = -1/Z_{12}$ $\qquad$ $Y_{11} = 1/Z_{12} + 1/Z_{13} + j0.03 + j0.025$

$Y_{13} = Y_{31} = -1/Z_{13}$ $\qquad$ $Y_{22} = 1/Z_{12} + 1/Z_{23} + 1/Z_{24} + 1/Z_{25} + j0.03 + j0.02 + j0.025 + j0.01$

$Y_{23} = Y_{32} = -1/Z_{23}$ $\qquad$ $Y_{33} = 1/Z_{13} + 1/Z_{34} + j0.025 + j0.015 + 1/Z_{23} + j0.02$

$Y_{24} = Y_{42} = -1/Z_{24}$ $\qquad$ $Y_{44} = 1/Z_{24} + 1/Z_{34} + 1/Z_{45} + j0.025 + j0.015 + j0.02$

$Y_{25} = Y_{52} = -1/Z_{25}$ $\qquad$ $Y_{55} = 1/Z_{25} + 1/Z_{45} + j0.01 + j0.02$

$Y_{34} = Y_{43} = -1/Z_{34}$

$Y_{45} = Y_{54} = -1/Z_{45}$

$Y_{11} = 1/Z_{12} + 1/Z_{13} + j0.03 + j0.025$

Bus admittance matrix (Y_{bus})

6.25 - j 18.695	-5.00 + j 15.00	-1.25 + j 3.75	0	0
-5.00 + j 15.00	**12.9167 - j38.6650**	**-1.6667 + j5.0000**	**-1.2500 + j3.7500**	**-5.0000 + j15.0000**
-1.2500 + j3.7500	**-1.6667 + j5.0000**	**5.4167 - j16.1900**	**-2.5000 + j7.5000**	**0**
0	**-1.2500 + j3.7500**	**-2.5000 + j7.5000**	**5.4167 - j16.1900**	**-1.6667 + j5.0000**
0	**-5.0000 + j15.0000**	**0**	**-1.6667 + j5.0000**	**6.6667 - j19.9700**

Now assume that a 0.5 per unit shunt resistance is added at bus 3 (i.e., on each phase a 0.5 per unit resistance is connected phase to ground). Calculate the new value of y_{33}:

y_{33} (new) = y_{33}(old) + (1/0.5) = **7.4167 -16.1900i** Do any other values of Ybus change? **No**

Now assume that a 75 Mvar (three phase) shunt capacitance is added at bus 4. Calculate the new value of y_{44}:

Since no transformer is present on any side of the network, we can assume voltage at all buses (bus 4 inclusive) to be 1.0 p.u. Also, in per unit, Q_c = j0.75, and from $Q_c = V^2 * B$, B = j0.75

y_{44} (new) = y_{44} (old) + j0.75 = **5.4167- j15.44**

5. Using PowerWorld Simulator and the case ECEN_615_HW1, give the bus numbers and circuit of two transmission lines that when opened cause at least one other line to be overloaded.

Among different, possible transmission line outages are:

Example Case	Disconnected		Circuit	Overloaded line		Circuit	Percent
	From	To	Num	From	To	Num	Line Loading
1	Pecan69	Pear 69	1	Pecan69	Pear 69	3	148
	Pecan69	Pear 69	2				
2	Cedar69	Olive69	2	Cedar69	Olive69	1	135
	Redbud69	Cherry69	1				
3	Peach69	Poplar69	1	Oak69	Walnut69	1	106
4	Elm345	Slack345	1	Elm138	Lemon138	1	102
	Elm138	Peach138	1				
5	Birch69	Lemon69	1	Cedar69	Olive69	1	114
	Peach69	Redbud69	1	Cedar69	Olive69	2	114